Permeability of Soils.

Soil is a particulate material and has pores that provides a passage for water. Such passages vary in size and are interconnected. The permeability of a soil is a property which describes quantitatively, the ease with which water flows through that soil.

Remember:

- The same soil may exhibit different degrees of permeability, depending upon its structure.
- A clay soil with a flocculated structure is more permeable than the same soil with a dispersed structure.

* Darcy's Law.

The flow of free water through soil is governed by Darcy's Law. Darcy established that the flow occurring per unit time is directly proportional to the head causing flow and the area of cross-section of the soil sample but is inversely proportional to the length of the sample i.e

$$q \propto \frac{\Delta h}{L} \cdot A$$

$$q = k \cdot \frac{\Delta h}{L} \cdot A = k \cdot i \cdot A.$$

Δh = head causing flow

Where, $i = \frac{\Delta h}{L}$, hydraulic gradient

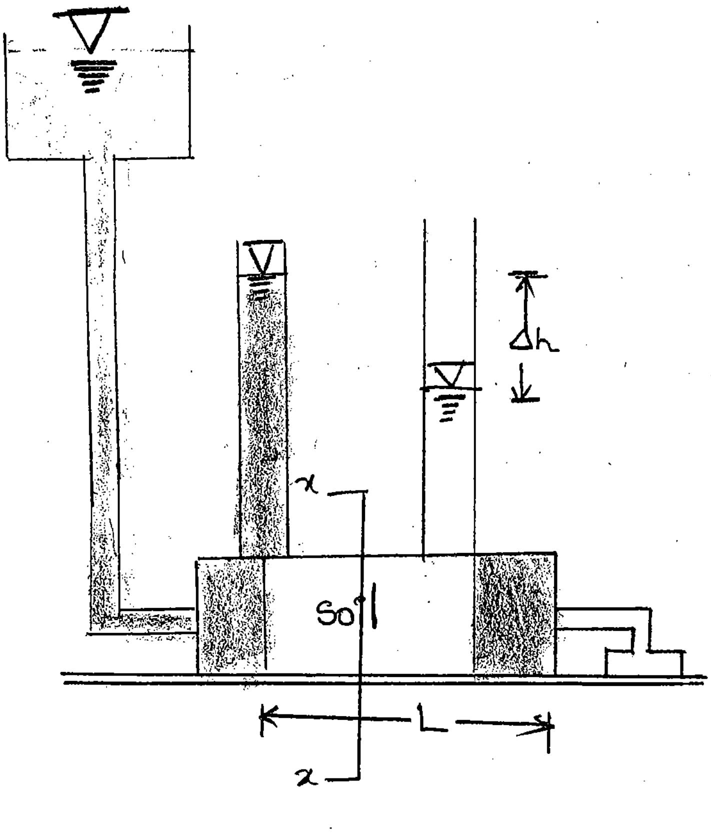
k = Coefficient of permeability of soil

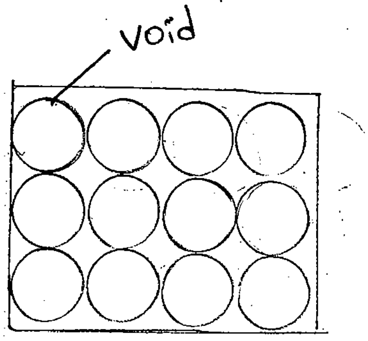
A = Area of cross-section.

Note: The coefficient of permeability of soil (k) is defined as the average velocity of flow which will occur under unit hydraulic gradient.

It has the unit same as velocity i.e cm/sec or m/day

* Discharge Velocity





Section - x - x.

Area of soil specimen at section X – X = A

From Darcy's Law

$$q = K.i.A$$

$$\frac{q}{A} = K.i = V.$$

Here, V is average velocity based on gross area of cross section. The average velocity is also referred as Superficial velocity. It is superficial because the actual flow is through pores in the cross section and not through entire cross-sectional area.

* Seepage Velocity

The flow through soils, however occurs only through the interconnected pores. The velocity through the pore is called Seepage velocity (V_s).

As the flow is continous, discharge q must be same throughout the system. Thus

$$q = A.V = A_v.V_s$$

$$V = \frac{A_v}{A}.V_s$$

Where A_v = area of voids in total cross-sectional area A.

Since $\frac{A_v}{A} \simeq \frac{V_v \text{ (volume of void)}}{V \text{ (Total volume)}} = n$ [$\because$ n = porosity]

$\therefore \quad V = n.V_s$

i.e seepage velocity $V_s = \frac{V}{n}$

Remember: Since $n<1$, the seepage velocity is always greater than the discharge or superficial velocity.

* Limitations of Darcy's Law

- Flow should fulfill continuity conditions
- The flow should be steady state laminar and one dimensional
- Soil should be completely saturated.
- No volume changes should occur during or as a result of flow.

* Coefficient of Permeability:

The coefficent of permeability can be defined using Darcy's equation.

$$q = k.i$$

Hence, the coefficent of permeability may be defined as the velocity of flow which would occur under unit hydraulic gradient. It is also called coefficent of hydraulic conductivity. Coefficent of permeability has dimension of velocity.

Typical value of the coefficent of permeability

S No	Soil Type	Coefficent of Permeability (cm/sec)	Drainage properties
1	Clean gravel	>1	Very Pervious
2	Sand	1×10^{-3} cm/s to 1cm/s	pervious
3	Silt	1×10^{-7} cm/s to 1×10^{-3} cm/s	poorly pervious
4	Clay	$< 10^{-7}$ cm/s	Imprevious

Remember:

The coefficent of permeability divided by porosity of soil is known as Coefficent of percolation (k_p).

$$K_p = \frac{K}{n}$$

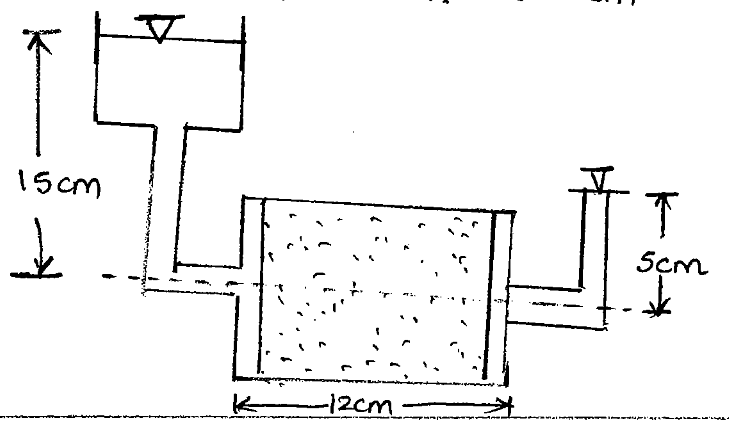
Where

k_p = Coefficent of percolation

K = Coefficent of permeability

n = porosity

1. For the soil specimen shown in figure. calculate the soil permeability if rate of flow is 2cc/min. Take area cross-section of soil specimen as $8cm^2$



Solution:

Given $L = 12cm$

$A = 8cm^2$

$Q = 2cc$ per minute $= \frac{2}{60}$ cc/sec

$H_L = H_1 - H_2 = 15 - 5 = 10cm.$

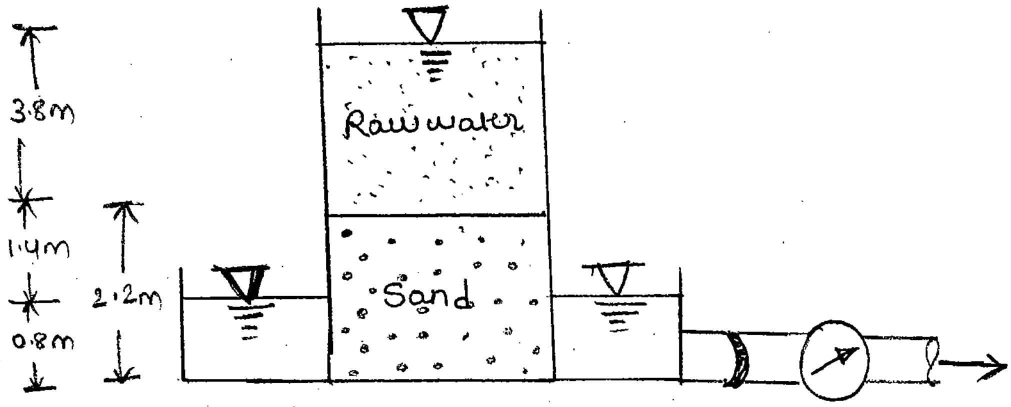
Hydraulic gradient $i = \frac{H_i}{L} = \frac{10}{12}$

Using Darcy's equation $Q = K.i.A.$

$$\frac{2}{60} = K.\frac{10}{12}.8$$

$$K = 5\times10^{-3} cm/sec.$$

2. A slow sand filter of area $2.85m^2$ consist $2.2m$ sandy layer as shown in figure. Calculate the amount of water supplied per day if permeability of sand is 4.8×10^{-5} m/s.



Solution: Given,

$A = 2.85m^2$

$K = 4.8\times10^{-5} m/s.$

Let us assume that datum is at bottom of filter.

∴ Headloss,

ΔH = Total available at top of sandlayer − total head after filtration through sand layer.

$= (2.2m + 3.8m) - (0 + 0.8m)$

$= 5.2m.$

∴ Hydraulic gradient $i = \dfrac{\Delta H}{\text{thickness of sandlayer}} = \dfrac{5.2\,m}{2.2m} = 2.364$

From Darcy's Law $Q = K.i.A.$

$= 4.8 \times 10^{-5} \times 2.364 \times 285$

$= 3.234 \times 10^{-4}\, m^3/s.$

∴ Discharge in a day $= Q \times 24 \times 60 \times 60\ m^3/d$

$= 3.234 \times 10^{-4} \times 86400\, m^3/d.$

$= 24.94\ m^3/d.$

3. Calculate the coefficent of permeability of soilsample 6cm in height and $50cm^2$ in cross sectional area, if a quantity of water equal 450ml passed down in 10 minutes under an effective constant head of 40cm. On oven drying, the test specimen weight 495g. Taking the Sp.g of soil solids as 2.65, calculate the seepage velocity of water during the test.

Solution: $A = 50cm^2$

$Q = 450ml / 10min = 0.45l / 600 sec$

$= \dfrac{0.45 \times 1000 cm^3}{600 sec} = 0.75 cm^3/sec.$

$$i = \frac{\Delta H}{L} = \frac{40cm}{6cm} = 6.67.$$

From Darcy's Law.

$$Q = K.i.A$$

$$0.75\,cm^3/s = K \times 6.67 \times 50cm^2.$$

$$K = \frac{0.75}{6.67 \times 50} = 2.25 \times 10^{-3}\ cm/sec.$$

Note that on oven drying, the volume of sample remain the same.

$$\gamma_d = \frac{W_d}{V} = \frac{495gm}{(50 \times 6)cm^3} = 1.65 gm/cm^3.$$

Using, $\gamma_d = \frac{G\gamma_w}{1+e}$

$$1.65 = \frac{1 \times 2.65}{1+e}$$

$$e = 0.606.$$

Using $n = \frac{e}{1+e}$

$$n = \frac{0.606}{1+0.606} = 0.377.$$

Now Seepage velocity $(V_s) = \frac{\text{Discharge velocity}}{\text{Porosity}(n)}$

Where Discharge velocity $(V) = \frac{\text{Discharge}(Q)}{\text{Area of Specimen}(A)}$

$$V = \frac{0.75cm^3/s}{50cm^2} = 0.015\,cm/s.$$

∴ Seepage velocity $(V_s) = \frac{0.015}{0.377}$ cm/s.

$$= 0.0398\,cm/s \text{ or } 3.98\,m/s$$

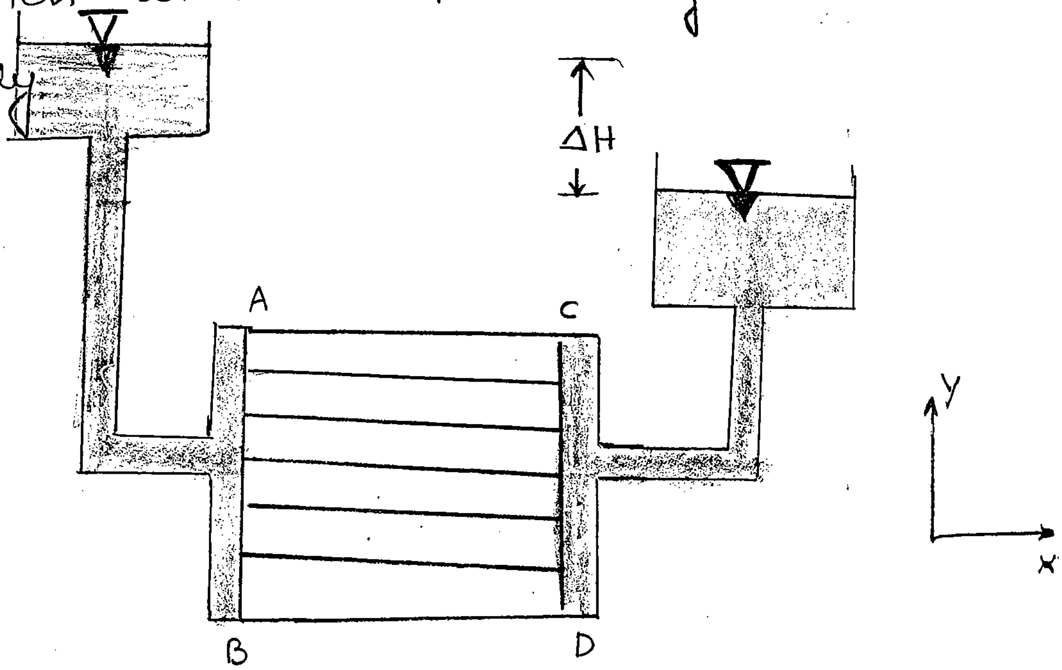
* Permeability of Stratified Soils.

In nature, soils are usually deposited in successive layers. Even if the different layers of the deposit are homogenous within themselves, this may lead to a considerable disparity in the average permeability parallel to the bedding plant and that normal to the bedding planes.

Broadly stratification can be considered as horizontal and continous, average coefficent for flow in horizontal and vertical directions can be estimated.

* Horizontal flow

Consider the solid profile, shown in figure below consisting of n number of layers with $H_1, H_2, \ldots H_n$ in thickness with their permeability $K_1, K_2, \ldots K_n$ respectively.



Let H = Total thickness of deposit

$= H_1 + H_2 + H_3 + H_4 \ldots\ldots H_n$

K_H = Average permeability in the horizontal direction

Assume the total head along the line AB to be constant. Similarly, the total head along CD may also be taken as constant. but value will be less than that along AB.

$\therefore \; i = i_1 = i_2 = i_3 = i_4 \ldots\ldots i_n$

Total discharge through the soil deposit = sum of discharges through the individual layers

$\therefore \; q = q_1 + q_2 + q_3 + q_4 + \ldots\ldots q_n$

$K_H . i . A = K_1 [A_1 + K_2 i A_2 + K_3 i A_3 + K_4 i A_4 + \ldots\ldots K_n i A_n$

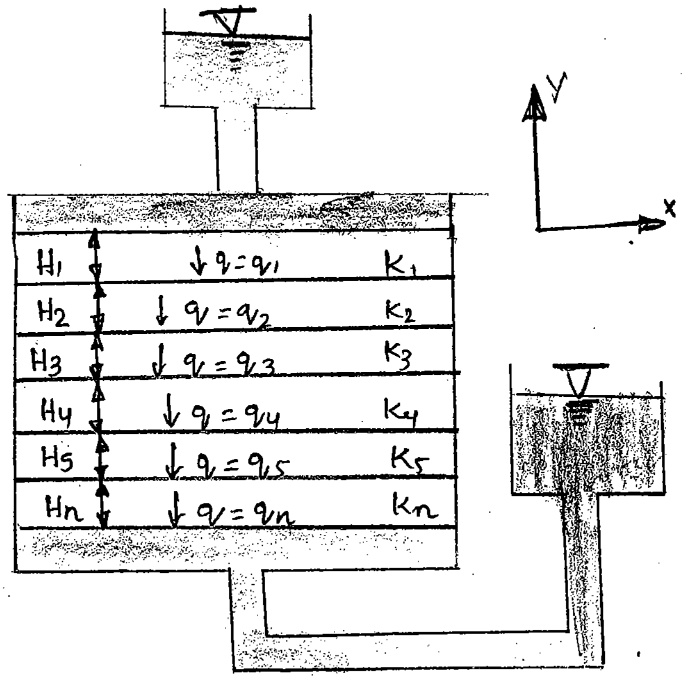
$\Rightarrow \; K_H . H = K_1 . H_1 + K_2 H_2 + K_3 H_3 + K_4 H_4 + \ldots\ldots K_n H_n$

$$K_H = \frac{K_1 H_1 + K_2 H_2 + K_3 H_3 + \ldots\ldots K_n H_n}{H}$$

$$K_H = \frac{K_1 H_1 + K_2 H_2 + K_3 H_3 + \ldots\ldots + K_n H_n}{H_1 + H_2 + H_3 + H_4 \ldots\ldots + H_n}$$

$$K_H = \frac{\Sigma K H}{\Sigma H}.$$

* Vertical Flow:



In this case, flow take place in the direction perpendicular to the stratification.

In this case, to satisfy the continuity condition.

$$q = q_1 = q_2 = q_3 = q_4 \cdots q_n$$

Let head loss through different layers be $h_1, h_2, h_3 \cdots h_n$

$\therefore$ Total head loss,

$$\Delta H = h_1 + h_2 + h_3 + h_4 + \cdots h_n \quad \text{--- (i)}$$

Also let K_V be the average permeability in vertical direction

Now, $\quad q = K_V . i . A = K_V . \frac{\Delta H}{H} . A$

or $\quad \Delta H = \frac{qH}{K_V . A}$

similarly, $\quad h_1 = \frac{qH_1}{K_1 A}, \; h_2 = \frac{qH_2}{K_2 A}$

$h_3 = \frac{qH_3}{K_3 A}, \; h_4 = \frac{q.H_4}{K_4 . A}$

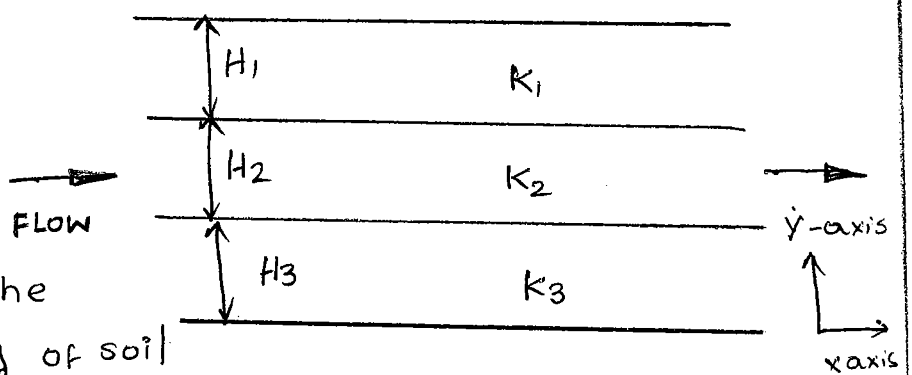
From equation (i) we get

$$\boxed{\frac{q.H}{K_V . A} = \frac{q.H_1}{K_1 A} + \frac{qH_2}{K_2 A} + \frac{qH_3}{K_3 A} + \frac{qH_4}{K_4 A} + \cdots \frac{qH_n}{K_n}}$$

or $\quad K_V = \dfrac{H}{\frac{H_1}{K_1} + \frac{H_2}{K_2} + \frac{H_3}{K_3} + \cdots \frac{H_n}{K_n}}$

1. A horizontal stratified soil deposit consist of three layers, each uniform in itself. The permeability of the layer are 8×10^{-4}, 50×10^{-4} and 15×10^{-4} cm/sec. and their thicknesses are 6m, 3m and 12m respectively. Find the effective average permeability of the deposit in horizontal and vertical directions.

Solution:



K_x be the meability of soil deposit in the direction flow

$$K_x = \frac{K_1H_1+K_2H_2+K_3H_3+\ldots}{H_1+H_2+H_3}$$

Using the above values, we have

$$K_x = K_H = \frac{8\times10^{-4}\times 6+50\times10^{-4}\times3+15\times10^{-4}\times12}{6+3+12}$$

$$= \frac{378}{21}\times10^{-4} = 18\times10^{-4}\ \text{cm/sec}$$

Permeability in vertical direction, K_y, being the permeability of soil deposit in the direction of flow

$$K_y = K_V = \frac{H_1+H_2+H_3}{\frac{H_1}{K_1}+\frac{H_2}{K_2}+\frac{H_3}{K_3}}$$

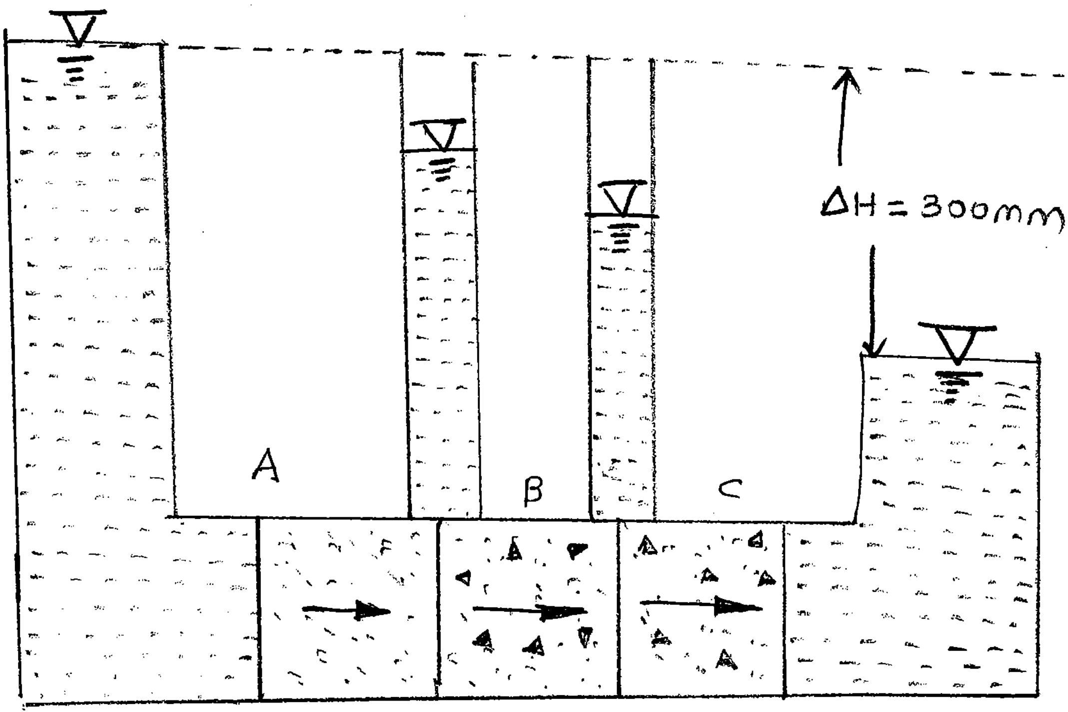
Using the above values, we get

$$k_y = K_V = \frac{6+3+12}{\frac{6}{8\times10^{-4}} + \frac{3}{50\times10^{-4}} + \frac{12}{15\times10^{-4}}}$$

$$= \frac{21\times10^{-4}}{0.75\times0.06*0.80} = \frac{21}{1.61}\times10^{-4}$$

$$= 13.04\times10^{-4}\,cm/sec.$$

2. The soil layers below have a cross-sectional area of 100mm x 10mm each. The permeability of each soil is $K_A = 10^{-2}$ cm/sec, $K_B = 3\times10^{-3}$ cm/sec, $K_C = 4.9\times10^{-4}$ cm/s. Find the rate of water supply in flow.



Sol: This is trick drawing it looks like a horizontal flow, but in reality its is a vertical flow because flow has to cross through every layer. Hence every layer has the same velocity.

$$V = V_1 = V_2 = V_3.$$

$$\therefore K_{eq} = \frac{H}{\frac{H_1}{K_1} + \frac{H_2}{K_2} + \frac{H_3}{K_3}} = \frac{450}{\frac{150}{1\times10^{-2}} + \frac{150}{3\times10^{-3}} + \frac{150}{4.9\times10^{-4}}}$$

$$\boxed{K_{eq.} = 1.2\times10^{-3}\ \text{cm/sec.}}$$

Head loss during flow $H_L = 300\text{mm}$

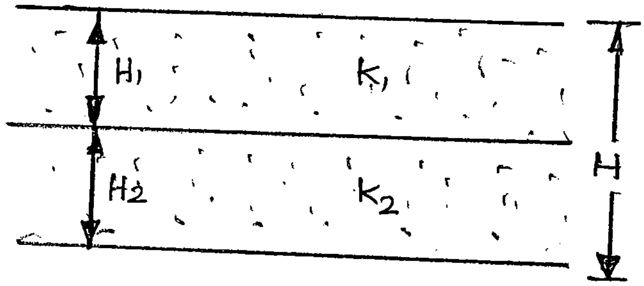
$\therefore$ Hydraulic gradient $i = \frac{H_2}{H} = \frac{300}{450} = \frac{2}{3}.$

By Darcy's equation

$$Q = K_{eq}.\,i.A = 1.2\times10^{-3} \times \frac{2}{3} (10\text{cm} \times 10\text{cm}) \times \frac{3600}{1}\ \text{cm}^3/\text{hr}$$

$$\boxed{Q = 291\ \text{cm}^3/\text{hr.}}$$

3. Prove that for Stratified deposits of soil, the average permeability in the horizontal direction is greater than the average permeability in the Vertical direction.



Sol: Consider a stratified deposists consisting two layers of thicknesses H_1 and H_2 with coefficent of permeability K_1 & K_2 as shown in figure.

We know average permeability in Horizontal and Vertical direction are given by

$$K_H = \frac{k_1 H_1 + k_2 H_2}{H_1 + H_2} = \frac{K_1 H_1 + K_2 H_2}{H}$$

$$K_V = \frac{H_1 + H_2}{\frac{H_1}{k_1} + \frac{H_2}{k_2}} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2}}$$

$[\because H = H_1 + H_2]$

Therefore $$\frac{K_H}{K_V} = \frac{(k_1 H_1 + k_2 H_2 / H)}{H / [(H_1/k_1) + (H_2/k_2)]}$$

$$= \frac{H_1^2 + H_2^2 [(k_1^2 + k_2^2) / k_1 k_2] H_1 H_2}{H^2}$$

$$= \frac{H_1^2 + H_2^2 + [[k_1^2 + k_2^2] / k_1 k_2] H_1 H_2}{H_1^2 + H_1^2 + 2 H_1 H_2}$$

Since k_1 and k_2 are always positive.

$$(k^1 - k^2)^2 \geq 0.$$

or $k_1^2 + k_2^2 \geq 2k_1k_2$

$$\frac{k_1^2 + k_2^2}{k_1 k_2} \geq 2.$$

The above ratio is equal to 2 only when $k_1 = k_2$ otherwise it is greater than 2. Here we take k_1 and k_2 different the above ratio will be greater than 2 always.

$\therefore$ $\frac{K_H}{K_V}$ in relation the nomerator is greater than denominator.

Hence $K_H > K_V$.

Determination Of Coefficient of Permeability

The coefficient of permeability of a soil can be determined using following methods.

(1) Laboratory methods

(2) Field methods

(3) Indirect methods.

1. Laboratory methods:

Laboratory tests can be performed on both undisturbed and remoulded samples. The coefficient of permeability can be determined in laboratory by the following methods.

(a) Constant head permeability test

(b) Variable head permeability test

(c) capillary permeability test.

(a) Constant head Permeability Test:

This test is preferred for coarse grained soils, such as gravels and sands. The complete set up for constant head permeameter is shown in figure.

An observation is taken by collecting a quantity of water in a graduated jar for a known time.

Let volume of water collected in time 't' is 'V'.

$\therefore$ Discharge $Q = \frac{V}{t}$

Also $K.\lambda.i.A = \frac{V}{t}$

[By Darcy's Law]

or $K.\frac{h}{L}.A = \frac{V}{t}$

$\left[\because i = \frac{HL}{L} = \frac{h}{L}\right]$

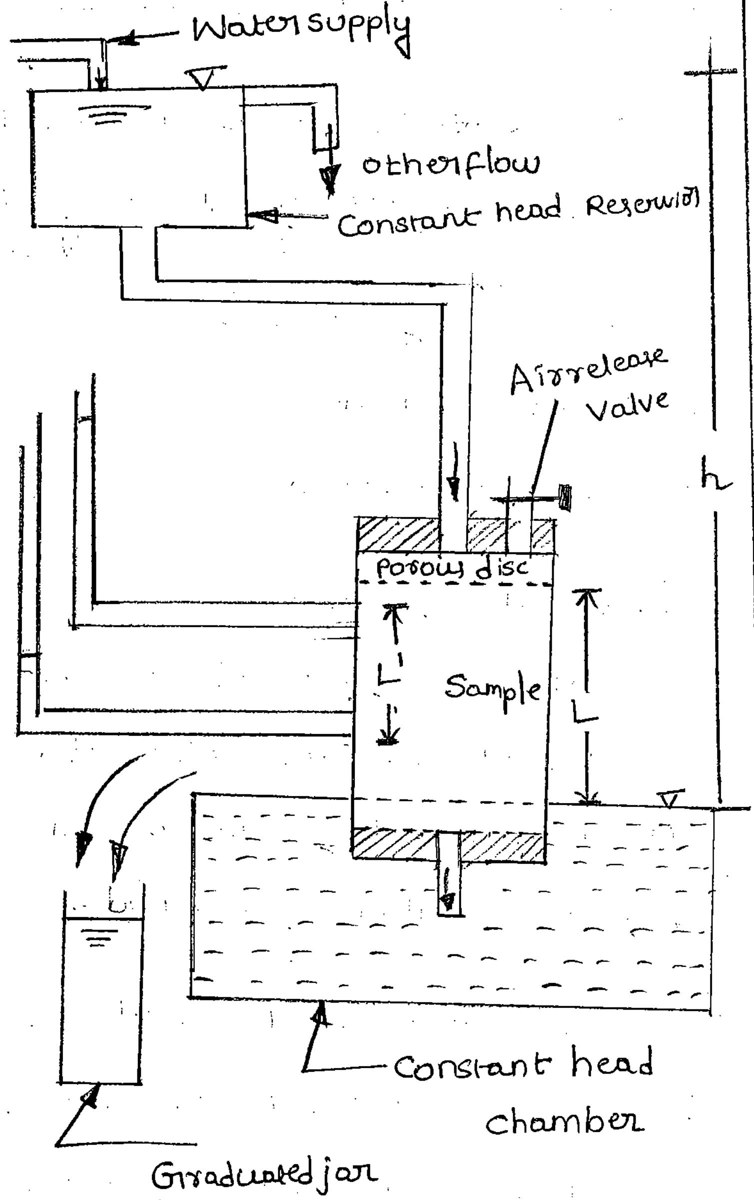
or $K = \frac{V.L}{A.t.h}$

Where,

L = distance between manometer taping point

A = Cross sectional area of sample

h = difference in monometric levels.



Constant head permeameter.

1. The following data were recorded in a constant head permeability test.

Internal diameter of permeameter = 7.5cm

Head lost over a sample length of 18cm = 24.7cm

Quantity of water collected in 60 seconds = 626ml.

Porosity of the soil sample = 44 %.

Calculate the coefficent of permeability of the soil. Also, determine the discharge velocity and the seepage velocity during the test.

Solution

Area of soil sample $A = \frac{\pi}{4} d^2$

$= \frac{\pi}{4} (7.5)^2 = 44.18\ cm^2$

$H = 24.7cm,\ L = 18cm,\ t = 60\,sec$

$\therefore$ Hydraulic gradient $i = \frac{H_L}{L}$

$= \frac{24.7}{18} = 1.372$

Discharge $Q = \frac{V}{t} = \frac{626}{60} = 10.433\ cm^3/s.$

From Darcy's equation $Q = K.iA$

$\Rightarrow 10.433 = K \times 1.372 \times 44.18.$

$K = 0.172\ cm/s.$

Discharging velocity $V = \frac{Q}{A}$

$$= \frac{10.433}{44.18} = 0.236 \text{ cm/s}$$

Given porosity of soil sample $n = 0.44$.

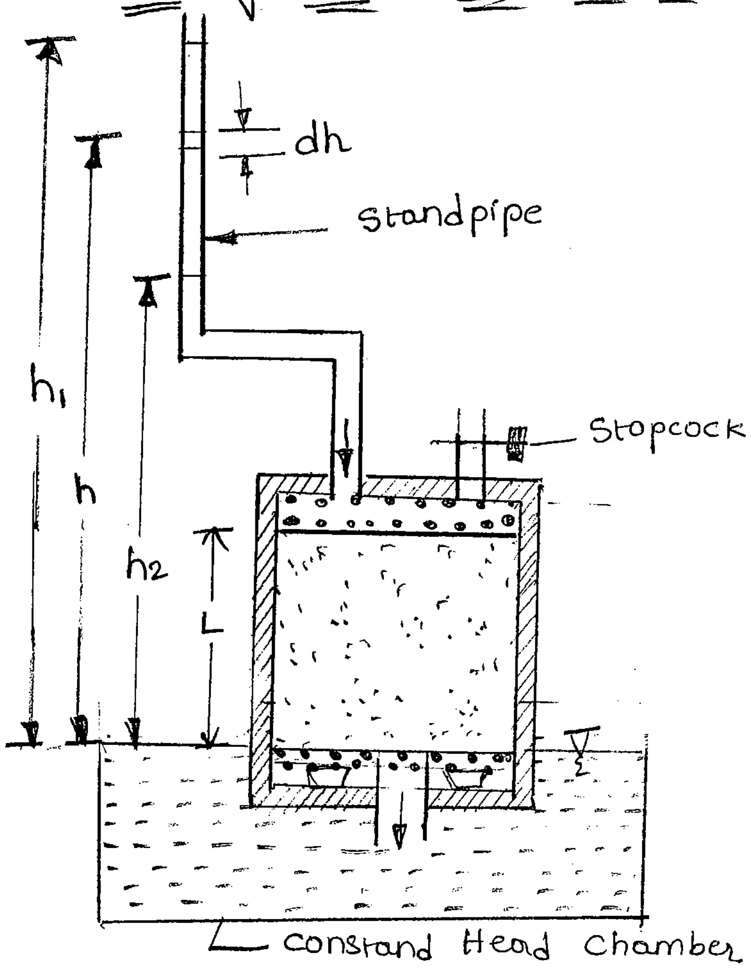
$\therefore$ Seepage velocity $V_s = \frac{V}{n} = \frac{0.236}{0.44} = 0.537$ cm/s.

(b) Variable head Permeability test:

Variable head permeability test is used for fine sands and silts which have relatively low permeability. The complete setup for variable head permeability test is shown in figure.

Falling Head Permeameter.

After saturation, the stand pipe of area cross-section 'a' is filled with water and time crosspending to h_1 is noted down. Now water is allowed to fall h_2 and time t_2 is noted.



Let at any intermediate stage water level be 'h' which falls by 'dh' in time "dt". So the head difference is h.

At that stage, let the discharge is "q"

$$q = K.i.A = K.\frac{h}{L}.A.$$

Volume of water collection in 'dt'

$$q.dt = a(-dh)$$

$$K.\frac{h}{L}.Adt = -adh$$

Integrating, $K.\int_{t_1}^{t_2} dt = \frac{-aL}{A}\int_{h_1}^{h_2}\frac{dh}{h}$

$$= \frac{aL}{A}\int_{h_2}^{h_1}\frac{dh}{h}.$$

$$K(t_2 - t_1) = \frac{aL}{A}(\log_e h_1 - \log_e h_2)$$

$$= \frac{aL}{A}\log_e\left(\frac{h_1}{h_2}\right)$$

$$K = \frac{aL}{At}\log_e\left(\frac{h_1}{h_2}\right)$$

Where $t = (t_2 - t_1)$

$$K = \frac{2.303\,aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right).$$

* Note *

- Let variable head test is performed in two stages on same soil, If, in first stage water level falls from h_1 to h_2 in time interval 't' and during second stage, waterlevel falls from h_2 to h_3 in same time interval 't' then h_1, h_2 and h_3 shall be related as

$$h_2 = \sqrt{h_1 h_3}$$

- The above results can be used to check the consistency of test in a soil. If variable head test is performed in two stages at equal time intervals, then above relation should hold true in each case.

2. In a falling head permeability test, the head causing fall was initially 90cm, and it drops 6cm in 15 minutes. How much time is required for the head to fall 45cm

Solution.

We know, the coefficent of permeability under falling head permeability test is given by

$$K = 2.303 \frac{aL}{At} \log_{10} \frac{h_1}{h_2}$$

Here $h_1 = 90cm$

$h_2 = 90 - 6 = 84cm$

and $t_1 = 15$ minutes

Therefore $t = 2.303 \frac{aL}{KA} \log_{10} \frac{h_1}{h_2}$ (or) $\frac{2.303 aL}{KA} = \frac{t}{\log_{10}\left(\frac{h_1}{h_2}\right)}$

$$\Rightarrow \frac{2.303 aL}{KA} = \frac{15}{\log_{10}\left(\frac{90}{84}\right)} = 500.61.$$

Let 'T' be the time interval in which head falls from 90cm ~~and~~ to 45cm

Therefore, $T = \frac{2.303 aL}{KA} \log_{10} \frac{h_1}{h_3}$

$$= 500.61 \times \log_{10} \frac{90}{45}$$

$$= 150.70 \text{ minutes.}$$

3. A permeameter of 100mm diameter with a sample length of 30cm was used for constant head and falling head test. While conducting a constant head test. the loss of head was 120cm for the soil sample and rate of flow was $3.2\ cm^3/s$. Find the coefficient of permeability. If a falling head test was performed on the same sample at the same void ratio, find the time taken for the head to fall from 98cm to 50cm. The diameter of stand pipe in the falling head test was 25mm.

Solution:

Constant head test

Area of sample $A = \pi\left(\frac{100^2}{4}\right) = 7854\ mm^2$

Hydraulic gradient $i = \frac{h}{L} = \frac{120}{30} = 4.$

Using $q = K.i.A.$

$\therefore \quad K = \frac{q}{i.A} = \frac{3.2\times10^3}{4\times7854} = 0.102\ m/s.$

Falling head test.

Area of stand pipe $a = \pi\left(\frac{25^2}{4}\right) = 490.9\ mm^2$

Using $K = 2.303\, aL \log_{10}\frac{h_1}{h_2}$

$$t = \frac{2.303aL}{KA}\log_{10}\frac{h_1}{h_2}$$

$$t = \frac{2.303\times490.9\times300}{0.102\times7854}\log_{10}\left(\frac{98}{50}\right).$$

(c) Capillary permeability test example.

4. A capillary permeability test was conducted in two stages under a head of 60cm and 180cm. In the 1st stage the wetted surface moved 1.5cm to 7cm in 7 minutes. In the second stage, it advanced from 7cm to 18.5cm in 24 minutes. The degree of saturation at the end of the test was 85%. and the porosity was 35%.

Determine the capillary head & coefficent of permeability.

Sol: Stage-I

$h_{o_1} = 60cm$

$h_{o_2} = 180cm$

$x_2 = 7cm$

$x_1 = 1.5cm$

$t_2 - t_1 = 7min$

Stage-II

$x_2' = 18.5cm$

$x_1' = 7cm$

$t_2' - t_1' = 24\ min$

$S = 85\%$

$n = 35\%$

For stage I,

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2K}{S.n}(h_{o_1} + h_c)$$

$$= \frac{7^2 - 1.5^2}{7} = \frac{2K}{0.85 \times 0.35}(60 + h_c)$$

$$\Rightarrow K(60 + h_c) = 0.9936. \quad \text{------} \quad (1)$$

Similarly for stage II,

$$\frac{x_2'^2 - x_1'^2}{t_2' - t_1'} = \frac{2K}{Sn}(h_{o2} + h_c)$$

$$\frac{18.5^2 - 7^2}{24} = \frac{2K}{0.85 \times 0.35}(180 + h_c)$$

$$\frac{7^2 - 1.5^2}{7} = \frac{2K}{0.85 \times 35}(180 + h_c)$$

$$K(180 + h_c) = 1.819. \quad \text{——} (2)$$

From equation (i) and (ii) we get

$$\frac{60 + h_c}{180 + h_c} = \frac{0.9936}{1.819}$$

$$1.819(60 + h_c) = 0.9936(180 + h_c)$$

$$109.14 + 1.819 h_c = 178.848 + 0.9936 h_c$$

$$0.8254 h_c = 69.708$$

$$h_c = 84.45 \text{ cm}$$

Substituting value of h_c in equation (i) we get

$$K(60 + 84.45) = 0.9936$$

$$144.45 K = 0.9936$$

$$K = 6.87 \times 10^{-3} \text{ cm/min.}$$

Principle of Effective Stress, capillarity

Terzaghi was the first to enunciate the effective stress principle. It can be said that in doing that, he opened the flood gates to the discipline of soil mechanics. Some of the basic developments on shear strength, compressibility and lateral pressures are a direct offshoot of the effective stress concept. The effective stress principle consists of two parts.

(i) Definition of the effective stress

(ii) Importance of effective stress in engineering behavior of soil.

* Total stress, Pore pressure and Effective Stress

Total stress

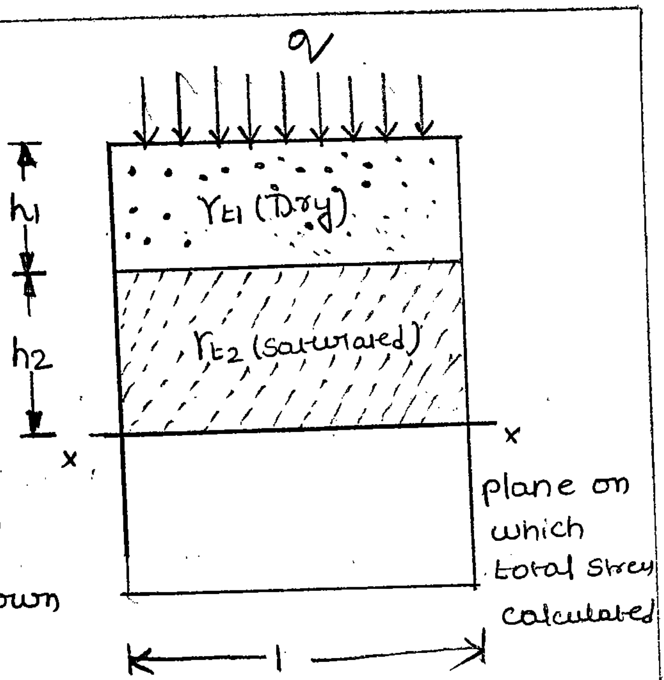
Normal stress is defined as the sum of the normal component of the forces (ΣN) over a plane ($x-x$) divided by the area of plane (A). Generally it is denoted by σ

$$\therefore \quad \sigma = \frac{\Sigma N}{A}$$

Let us consider a stratified soil sample of unit width and unit length for the plane over which normal stresses to be computed

$$\therefore\ \sigma = \frac{q.A + \gamma_{t_1} \times h_1 \times A \times \gamma_{t_2} \times h_2 \times A}{A}$$

$$= q + \gamma_d h_1 + \gamma_{sat} h_2$$

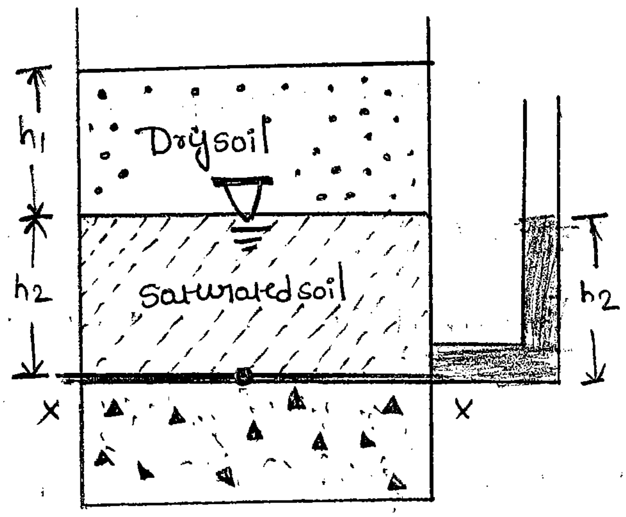


If external pressure q is zero then total stress called due to the over burden alone are known as "Geostatic stress."

then, $\sigma = \gamma_d h_1 + \gamma_{sat} h_2$

* Pore pressure:

It is the pressure exerted by the pore water filled in the void of the soil skeleton. It is denoted by u and equal to the depth (h_2) below the ground watertable upto the point (A) where it is measured and multiplied by the unit weight of water γ_w



$$u = h_2 \gamma_w \qquad \text{— (ii)}$$

Pore pressure is also called the neutral stress because it acts on all sides of the particle, hence doesnot cause the soil particles to press against adjacent particles.

Remember: Pore water pressure, like total stress is also a measurable parameter. A stand pipe, piezometer or pore water pressure transducer may be used to measure the pore water pressure.

* Principle of Effective stress:

Total stress (σ) is made up of two parts

(i) one part is due to pore water pressure (u)

(ii) the other part is due to pressure exerted by the soil skeleton which is called effective stress ($\bar{\sigma}$).

i.e $\quad \sigma = u + \bar{\sigma}$

$$\bar{\sigma} = \sigma - u$$

Substituting value of σ and u from equation (i) and (ii) we have

$$\bar{\sigma} = (\gamma_d h_1 + \gamma_{sat} h_2) - \gamma_w h_2$$

$$= \gamma_d h_1 + (\gamma_{sat} - \gamma_w) h_2 = \gamma_d h_1 + \gamma' h_2$$

Where γ' is submerged unit weight.

Remember:

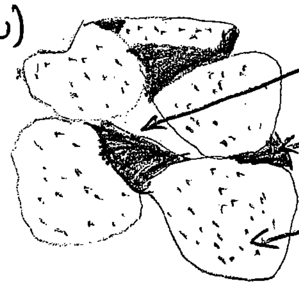
- Effective stress in soils is a grain to grain contact pressure which a soil particle can exhibit.
- The effective stress is not a physical parameter and cannot be measured.
- Increase in effective stress causes the particles to pack more closely, decreases the void ratio, leads to a

decrease in compressibility and increases in the shearing resistance of the soil.

- When there is an equal increases in the total stress and the pore pressure, then effective stress remain unchanged.
- Principle of effective stress applies only to normal stresses not to the shear stress.

* Effective stress in Partially Saturated Soils:

Effective stress $\bar{\sigma} = \sigma - u_a + \chi (u_a - u_w)$



Where σ = Total stress

u_a = Pore air pressure

u_w = Pore water pressure

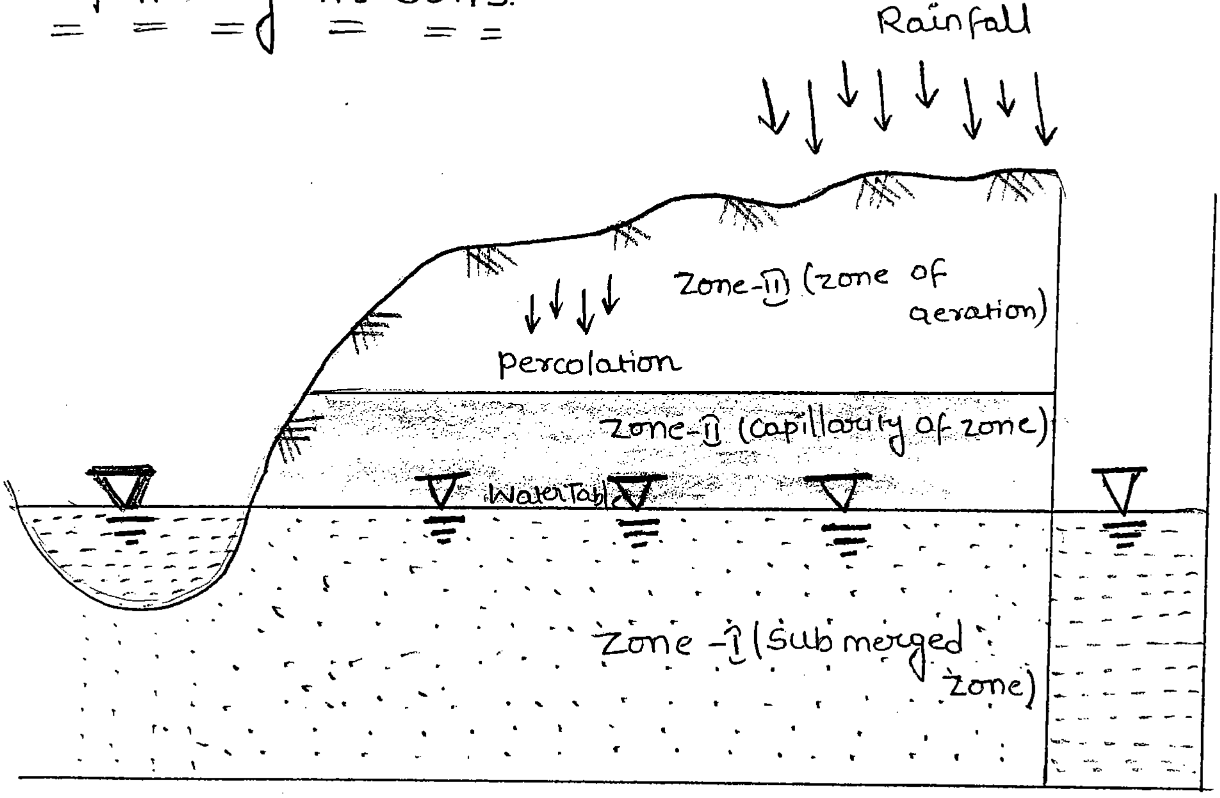
χ = The parameter to be determined experimentally

= 0 (for dry soil)

= 1 (for saturated soil)

It means that, if the soil is partially saturated, pore air pressure is considered along with pore water pressure to analyze the stresses in the soil mass.

Capillarity in Soils.



Zones of soil water below ground surface.

As we know, the rain water percolates into the ground under the influence of gravity and gets stored in the soil pores, over an impervious stratum, in the form of ground water reservoir.

The upper surface of the zone of full saturation of the soil is called the water table or phreatic surface. At the water table, the groundwater is subjected to atmospheric pressure. In other words, the pore pressure is zero.

Soil water can exist in three zones as shown below:

- Zone - I (submerged zone):

This zone exist below the ground water table. The soil in this zone is in submerged condition and pore pressure is hydrostatic i.e positive.

- Zone - II (capillarity zone):

If gravity was the only force, acting on the percolating water and taking it downward, then the soil above the water table would be completely dry But it is not in actual practice. Soil in this zone is completely saturated upto some height above the watertable. This phenomenon of rising water in soil is known as capillarity in soils.

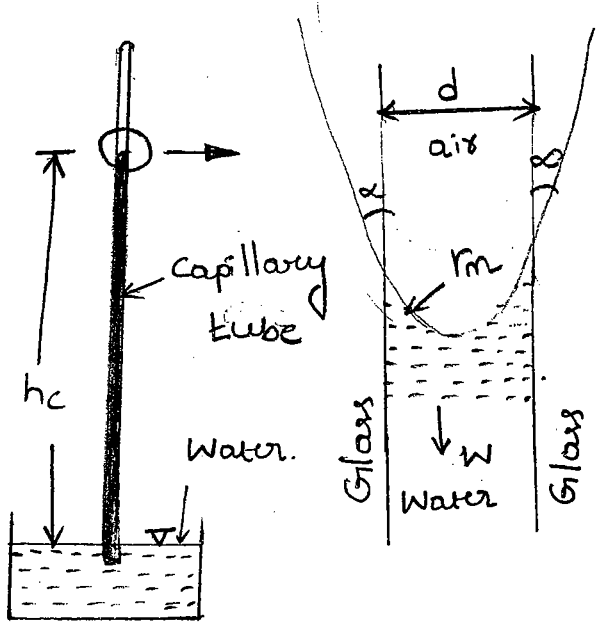
- Zone - III (Zone of Aeration):

Above the capillarity zone and upto a certain height, there exist a zone called the zone of Aeration in which the soil is able to retain small water droplets, surrounded on all sides by air. This water is known as contact moisture or hygroscopic water. The contact moisture is retained by the soil against the gravity drainage by the action of capillarity

Capillary Rise in Soils.

We can get an understanding of capillarity in soil by idealizing the continous void spaces as capillarity tubes. Consider a single idealized tube as shown in figure. below.

The surface tension (force) pulls the water upward till the height hc, at which the weight of water in the column is in equilbrium with the magnitude of the surface tension force.



From equilibrium

$$h_c \frac{\pi d^2}{4} \gamma_w = (\pi \cdot d) T\cos\alpha$$

Capillary Rise in soils.

$$h_c = \frac{4T\cos\alpha}{\gamma_w \cdot d}$$

Where α = contact angle

T = surface tension

d = diameter of capillary tube

γ_w = unit weight of water.

Remember:

- $h_c\,(cm) = \dfrac{0.3084}{d\,(cm)}$ .... at $4^\circ c$

- $h_c\,(cm) = \dfrac{0.2975}{d\,(cm)}$ .... at $20^\circ c$

Because of the complex nature of the soil, a theoretical prediction of capillary rise in soil is not possible.

Terzaghi and Peck suggested an approximate relationship to find capillary rise in-situ.

$$h_c\,(cm) = \frac{C}{eD_{10}}$$

Where C = Empirical constant, the value of which depends on the shape and surface impurite of the grains and lies between $0.1 - 0.5\,cm^2$.

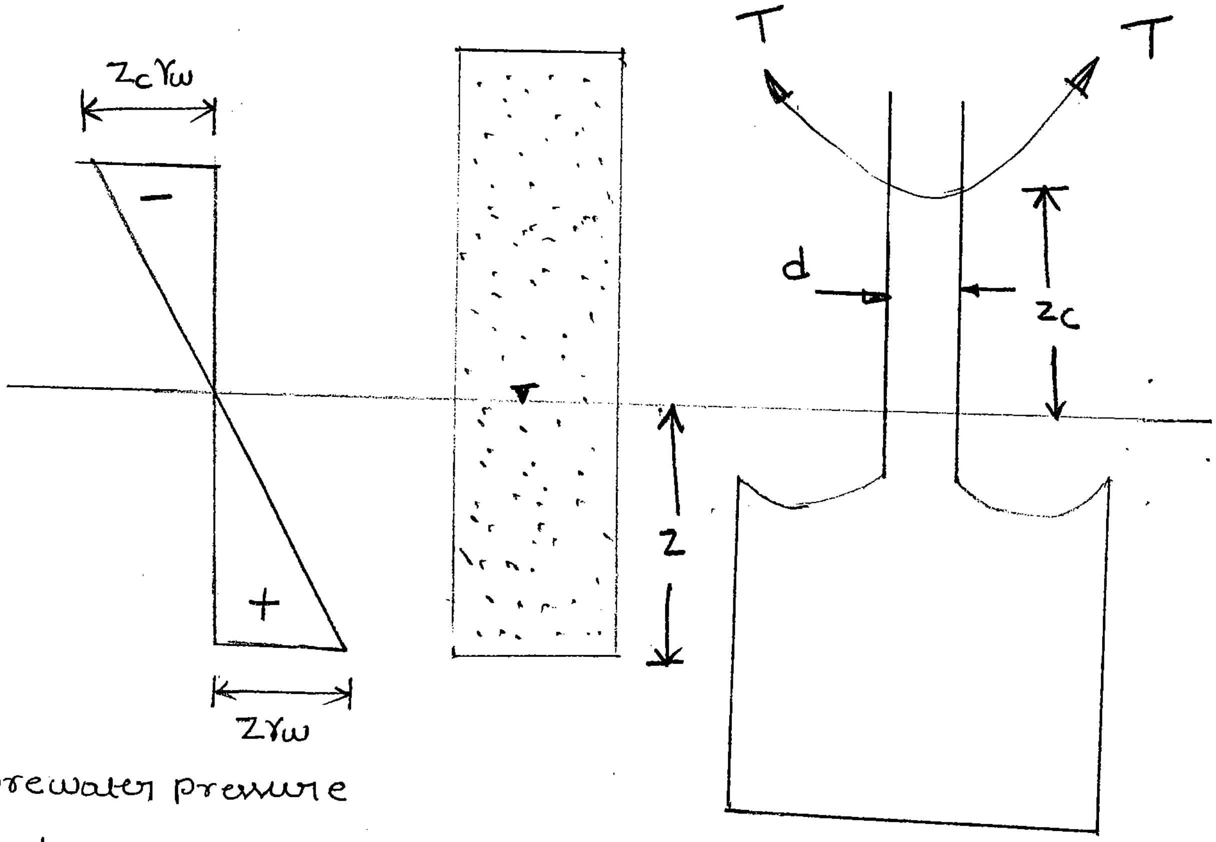
e = Void ratio

D_{10} = Effective grain size.

Do you know ?

Capillarity involves both adhesive and cohesive forces.

* Capillary Pressure



Porewater pressure distribution.

Idealization.

Capillary water rises against gravity and is held by the surface tension. Therefore the capillary water exerts a tensile force on soil and resulting negative pressure of water (capillary pressure) creates attraction between the particles.

From the effective stress equation

$$\bar{\sigma} = \sigma - u$$

When the pore water is in compression, as in case of hydrostatic pressure, u is taken positive whereas when it is in tension, u is taken negative.

Therefore effective stress in capillary zone

$$\bar{\sigma} = \sigma - (-u_c) = \sigma + u_c$$

It means that due to capillary in soils, effective stress increases.

Do you know?

- Negative pressure of water (capillary pressure) held above watertable results in attractive forces between particles. It is called as soil suction.
- Bulking of sand also occurs because of capillarity. Capillarity produces apparent cohesion which holds the particles in cluster, enclosing honey combs.
- Water may flow over the crest of an impermeable core in a dam even though, the free surface may be lower than the crest of core. This effect of capillarity is known as capillary siphoning.

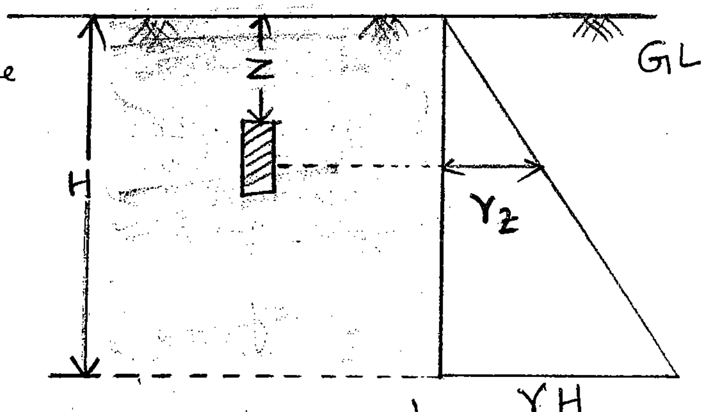
Geostatic Stresses in Soils.

Stresses within the soil mass may be caused by the self weight of the soil and also by the external loads which may be applied to the soil. The pattern of stresses caused by the external loads is usually complicated one. However, there is one common situation in which the self-weight of soil gives rise to a very simple pattern of stresses, that is when the ground surface is horizontal and the nature of soil does not very significantly in the horizontal direction. Such a situation frequently occurs in the case of sedimentary deposits. The total stress caused in such a situation at a point in a soil mass is called geostatic stress.

Do you know?

- In the geostatic situation, there are no shear stresses upon the horizontal or vertical planes within the soil.
- If external loading contributes to this stress, the stress will simply be called as the total stress.

The vertical geostatic stress at a point below the surface is equal to the weight of the soil lying directly above the point.



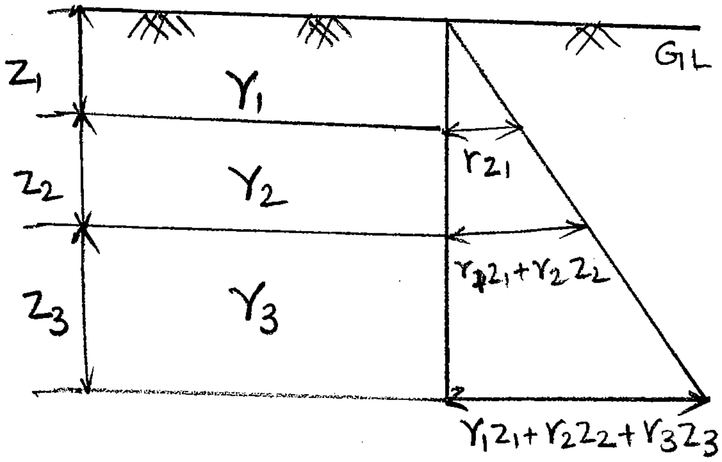
Uniform soil

If the unit weight of the soil (γ) is constant with depth,

then Total stress = σ

$$= \gamma (z \times 1 \times 1)$$

$$= \gamma \cdot z$$



Stratified soil.

If the soil is stratified with different unit weights for each stratum as shown in figure below.

Then σ can be calculated as

$$\sigma = \gamma_1 z_1 + \gamma_2 z_2 + \gamma_3 z_3 + \ldots\ldots$$

$$= \Sigma \gamma z.$$

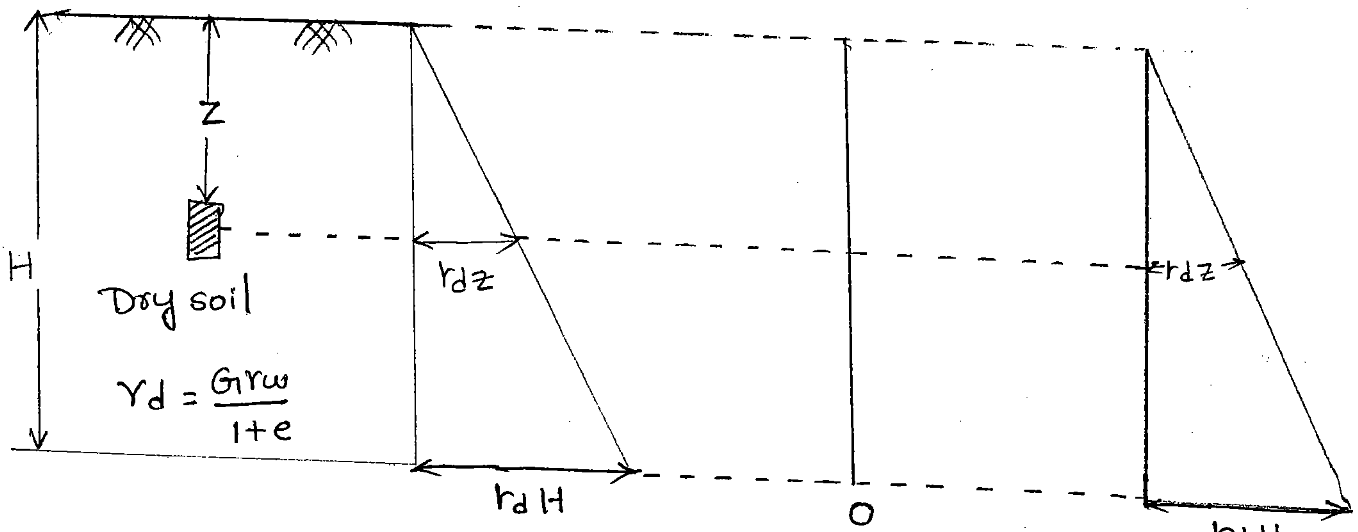
Case 1: When soil is Dry

Consider a situation when the watertable is at large distance from the ground surface. Then, the stresses at a depth z is given as

Total stress $\sigma = \gamma_d z$

Pore water pressure $u = 0$

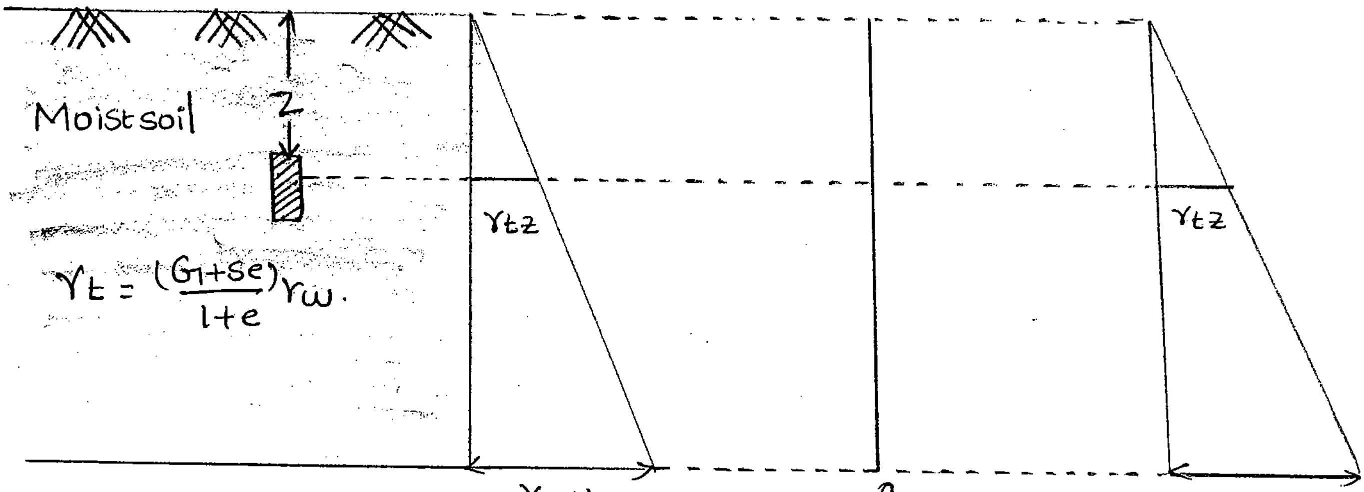
and effective stress $\bar{\sigma} = \sigma - u = \gamma_d z - 0 = \gamma_d z$



(a) Total stress (σ) (b) pore pressure (u) (c) Effective stress ($\bar{\sigma}$)

Vertical stress distribution in dry soil

Case : 2 When soil is Moist



(a) Total stress (σ)   (b) pore pressure (u)   (c) Effective stress ($\bar{\sigma}$)

Vertical stress distribution in moist soil

There is a situation of partially saturated soil wherein it is difficult to predict the pore water pressure distribution. In this case pore pressure is neglected. Bulk unit weight of soil is calculated by

$$\gamma_t = \frac{(G+Se)}{1+e}\gamma_w$$

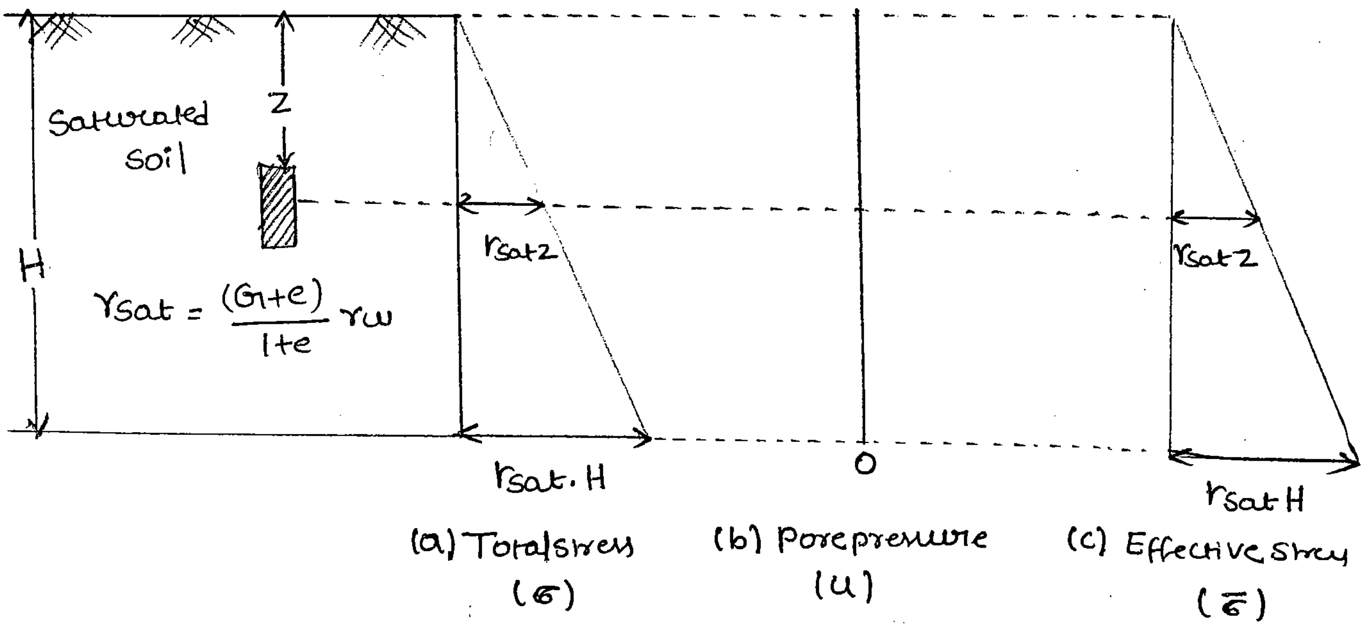
Thus

Total stress $\sigma = \gamma_t z$

pore water pressure $u = 0$

$$\bar{\sigma} = \sigma - u = \gamma_t z - 0 = \gamma_t z.$$

Case 3: When soil is saturated.



Vertical stress distribution in saturated soil.

This is a situation of saturated soil above water table, which is due to some reasons like rain, watering or irrigation. Hence pore water pressure is zero. Therefore, the stresses are governed by the saturated weight of the soil.

Thus,

Total stress $\sigma = \gamma_{sat} . z$

Pore water pressure $u = 0$

Effective stress $\bar{\sigma} = \sigma - u = \gamma_{sat} z - 0 = \gamma_{sat} . z$.

Case 4: When soil is completely submerged

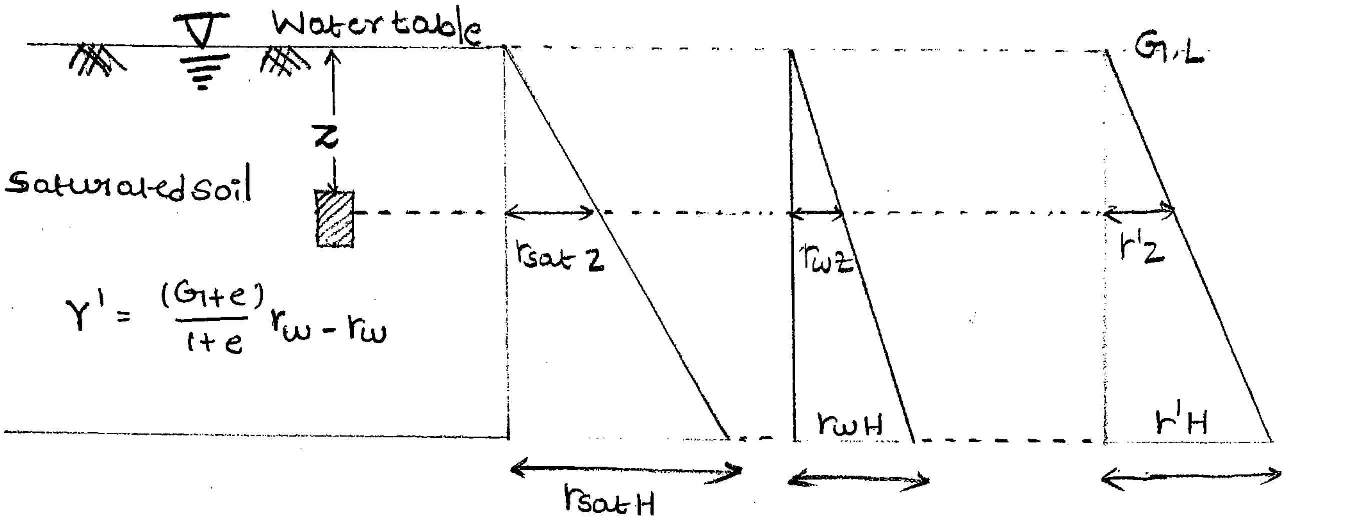
In this situation, soil is under the water table in submerged condition with watertable at groundlevel. Therefore pore, water pressure is hydrostatic. In this case, total stress is governed by saturated unit weight of soil and effective stress is governed by submerged unit weight.

Thus,

Total Stress $\sigma = \gamma_{sat} \cdot Z$

Pore water pressure $u = \gamma_w \cdot Z$

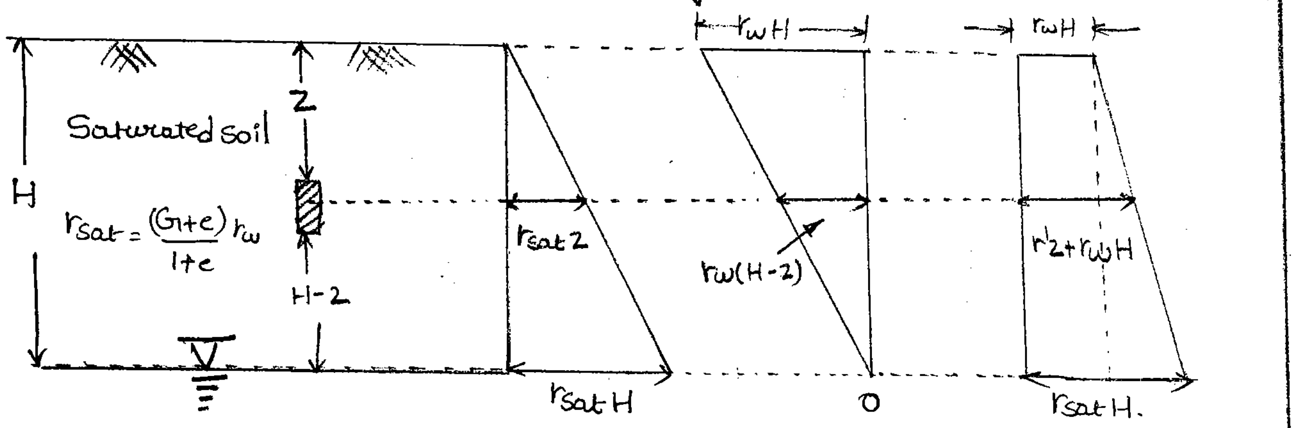
Effective stress $\bar{\sigma} = \sigma - u = \gamma_{sat} \cdot Z - \gamma_w Z = \gamma' Z$



(a) Total stress (σ) (b) Pore pressure (u) (c) Effective stress ($\bar{\sigma}$)

Vertical stress distribution in submerged soil

Case : 5 When soil is completely saturated by capillarity



(a) Total stress (σ) (b) pore pressure (u) (c) Effective stress ($\bar{\sigma}$)

Vertical stress distribution in saturated soil by capillarity.

In this situation, soil is completely saturated but porewater pressure is negative and vary linearly from zero at watertable to $\gamma_w(H-Z)$ at any depth below groundlevel.

Total stress $\sigma = \gamma_{sat} \cdot Z$

Pore pressure $u = \gamma_w(H-Z)$

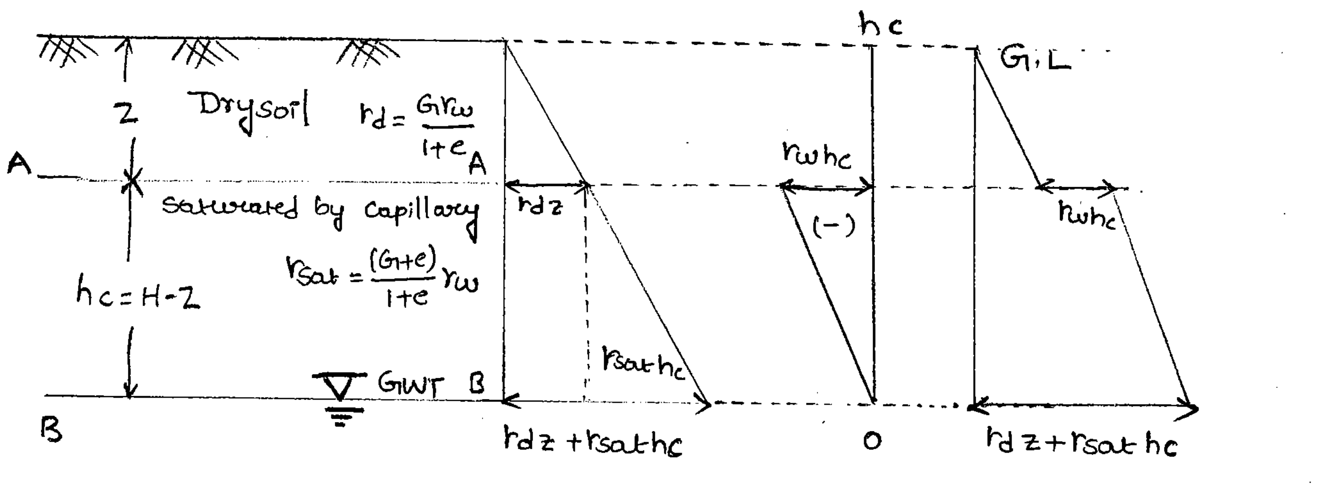
Effective stress $\bar{\sigma} = \sigma - u = \gamma_{sat} \cdot Z + \gamma_w(H-Z)$

$= (\gamma_{sat} - \gamma_w)Z + \gamma_w H.$

$= \gamma' Z + \gamma_w H.$

Special case:

When height of capillary rise in less than H. (i.e $0 < h_c < H$).



(a) Total stress (σ) (b) pore pressure (u) (c) Effective stress ($\bar{\sigma}$)

Vertical stress distribution in partially saturated soil

At Ground Level

Total stress $\sigma = 0$

Pore water pressure $u = 0$

Effective stress $\bar{\sigma} = 0$.

At the Level of A-A.

Total stress $\sigma_A = \gamma_d . z$

Pore water pressure $u_A = -\gamma_w h_c$

Effective stress $\bar{\sigma} = \sigma - u = \gamma_d z - (-\gamma_w h_c) = \gamma_d z + \gamma_w h_c$

At the Level of B-B

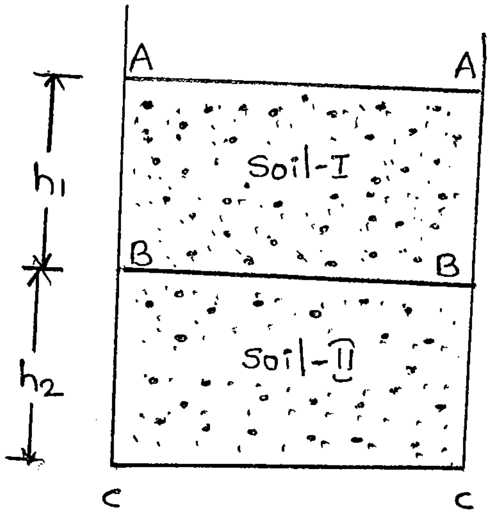
Total stress $\sigma_B = \gamma_d . z + \gamma_{sat} h_c$

Pore water pressure $u_B = 0$

Effective stress $\bar{\sigma} = \sigma - u = \gamma_d z + \gamma_w h_c$.

Effect of water table Fluctuations on Effective stress

Figure shows a layered soil system consisting of Soil-1 and Soil-2.



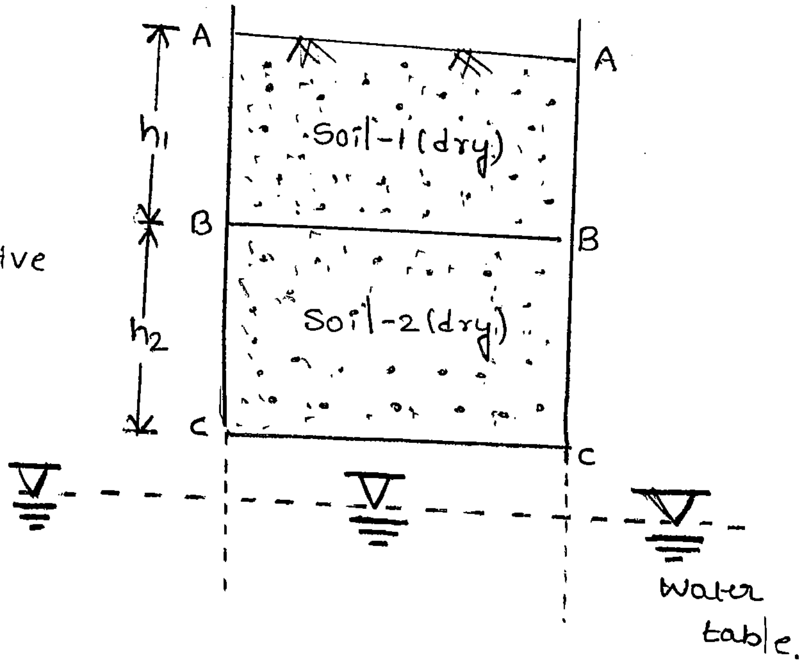
Case 1: When water table is below C-C:

(a) Total, Neutral and Effective stress at A-A:

$\sigma_A = 0$

$u_A = 0$

$\bar{\sigma} = \sigma_A - u_A = 0.$



(b) Total Neutral and Effective stress at B-B:

$\sigma_B = \gamma_{d_1} h_1$

$u_B = 0$

$\bar{\sigma} = \sigma_B - u_B$

$= \gamma_{d_1} h_1 - 0 = \gamma_{d_1} h_1$

(c) Total, Neutral and Effective stress at C-C

$\sigma_C = \gamma_{d_1} h_1 + \gamma_{d_2} h_2$

$u_C = 0.$

$\bar{\sigma} = \sigma_C - u_C = (\gamma_{d_1} h_1 + \gamma_{d_2} h_2) - 0 = \gamma_{d_1} h_1 + \gamma_{d_2} h_2$

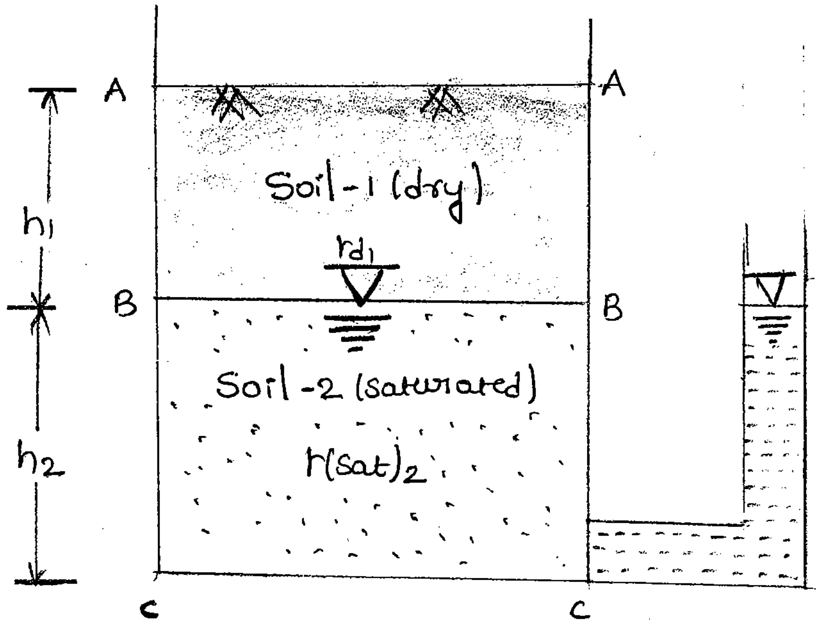
Case 2: When watertable is at B-B:

(a) Total, Neutral and Effective stress at A-A

$\sigma_A = 0$

$U_A = 0$

$\bar{\sigma}_A = \sigma_A - U_A = 0$



(b) Total, Neutral and Effective stress at B-B

$\sigma_B = \gamma_{d_1} h_1$

$U_B = 0$

$\bar{\sigma}_B = \sigma_B - U_B = \gamma_{d_1} \cdot h_1 - 0 = \gamma_{d_1} h_1$

(c) Total, Neutral and Effective stress at C-C:

$\sigma_c = \gamma_{d_1} h_1 + \gamma(sat)_2 \cdot h_2$

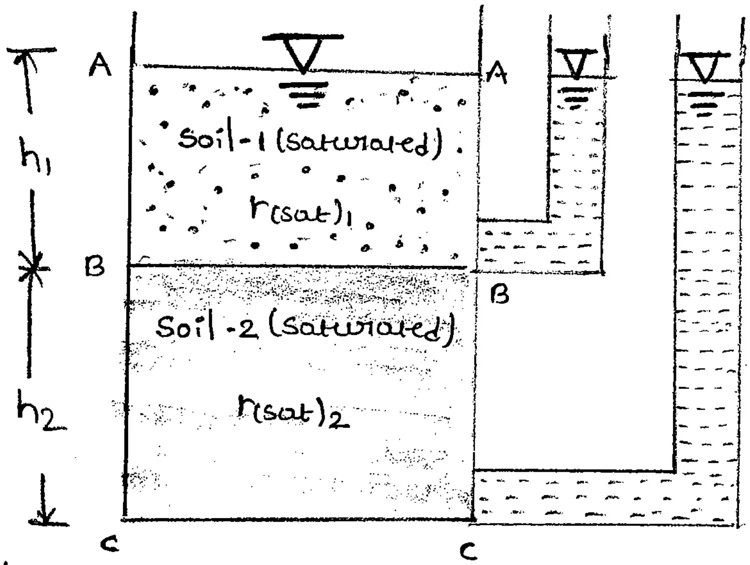
$U_c = \gamma_w \times h_2$

$\bar{\sigma}_c = \sigma_c - U_c = (\gamma_{d_1} \times h_1 + \gamma_{(sat)_2} \times h_2 - \gamma_w \times h_2)$

$= \gamma_{d_1} h_1 + \gamma_{sub} h_2.$

Case 3: When water table is at Ground level i.e A-A.

(a) Total, Neutral and Effective stress At A-A.



$$\sigma_A = 0$$

$$U_A = 0$$

$$\bar{\sigma}_A = \sigma_A - U_A = 0.$$

(b) Total, Neutral and Effective stress at B-B:

$$\sigma_B = \gamma_{(sat)_1} \cdot h_1$$

$$U_B = \gamma_w \cdot h_1$$

$$\bar{\sigma}_B = \sigma_B - U_B = \gamma_{(sat)_1} \cdot h_1 - \gamma_w h_1$$

$$= \left[\gamma_{(sat)_1} - \gamma_w\right] h_1$$

$$= \gamma_1' \cdot h_1$$

Where γ_1' = submerged unit weight for soil-I

(c) Total, Neutral and Effective stress at C-C:

$$\sigma_c = \gamma_{(sat)_1} \cdot h_1 + \gamma_{(sat)_2} \cdot h_2$$

$$U_c = \gamma_w (h_1 + h_2)$$

$$\bar{\sigma}_c = \sigma_c - U_c = \gamma_{(sat)_1} \cdot h_1 + \gamma_{(sat)_2} \cdot h_2 - \gamma_w (h_1 + h_2)$$

$$= \left[\gamma_{(sat)_1} - \gamma_w\right] \cdot h_1 + \left[\gamma_{(sat)_2} - \gamma_w\right] h_2$$

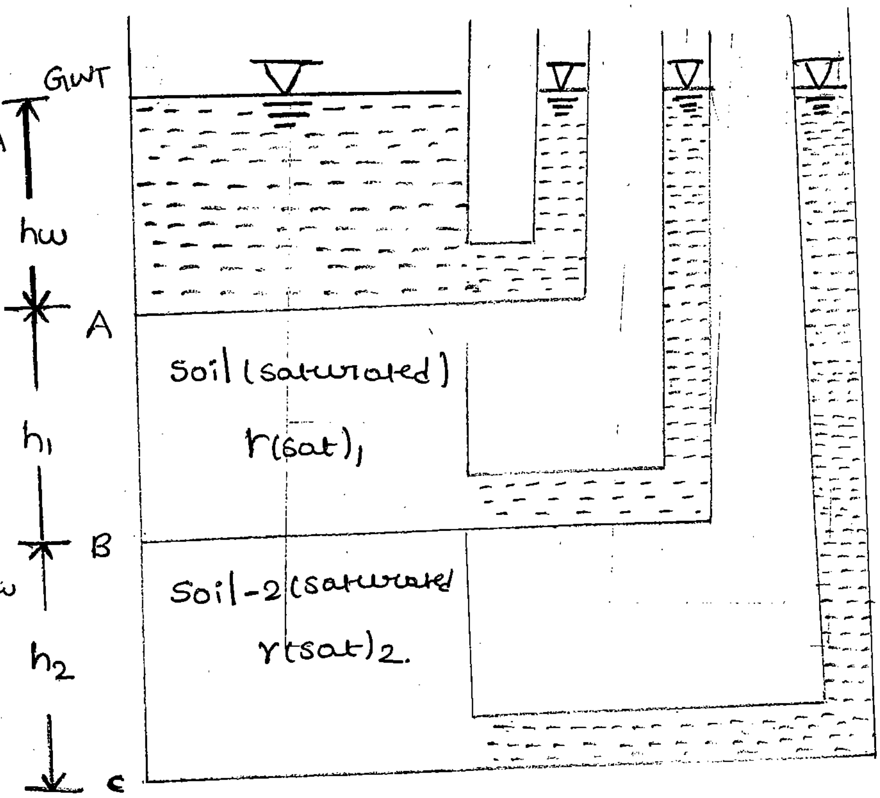
$$= \gamma_1' \cdot h_1 + \gamma_2' h_2$$

Where γ_1' = Submerged unit weight for soil-1

γ_2' = Submerged unit weight for soil-2

Case 4: When water table is above Groundlevel

(a) Total, Neutral and Effective stress at A-A



σ_A = Overburden pressure due to water column of h_w height $= \gamma_w \times h_w$

$U_A = \gamma_w \cdot h_w$

$\bar{\sigma}_A = \sigma_A - U_A = \gamma_w h_w - \gamma_w h_w$
$= 0.$

(b) Total, Neutral and Effective stress at B-B

$\sigma_B = \gamma_w . h_w + \gamma(sat)_1 . h_1 + (\gamma_{sat})_2 \cdot h_2$

$U_B = \gamma_w(h_w + h_1)$

$\bar{\sigma}_B = \sigma_B - U_B$

$= \gamma_w h_w + \gamma(sat)_1 . h_1 - \gamma_w(h_w + h_1)$

$= (\gamma_{sat})_1 - \gamma_w) \cdot h_1 = \gamma_1' \cdot h_1.$

(C) Total, Neutral and Effective stress at C-C:

$\sigma_C = \gamma_w . h_w + \gamma(sat)_1 . h_1 + \gamma(sat)_2 \cdot h_2$

$U_C = \gamma_w(h_w + h_1 + h_2)$

$\bar{\sigma}_C = \sigma_C - U_C$

$= \gamma_w \cdot h_w + \gamma(sat)_1 \cdot h_1 + \gamma(sat)_2 \cdot h_2 - \gamma_w(h_w + h_1 + h_2)$

$= [\gamma(sat)_1 - \gamma_w] . h_1 + [\gamma(sat)_2 - \gamma_w] h_2$

$= \gamma_1' . h_1 + \gamma_2' \cdot h_2.$

Where γ_1' = submerged unit weight for soil-1

γ_2' = submerged unit weight for soil-2.

* Seepage Through soils

Seepage is the flow of water under gravitational forces in a permeable medium. Flow of water take place from a point of high head to a point of low head. In one dimensional flow, where Darcy's law is valid, the velocity of flow is taken constant at every point over a cross section normal to the direction of flow. However many practical situations (like flow through earth dams, sheet piles) the flow of water is not unidirectional and velocity of water does not remain constant. In such cases, the quantity of seepage and other parameters such as hydraulic gradient and pore water pressure are estimated by the help of flow nets. The concept and construction of flow nets are based on Laplace's equation of continuity.

* Type of Head:

There are three types of head available in fluid flow.

(i) Velocity head

(ii) Pressure head

(iii) Datum or elevation head.

(i) Velocity head:

- It is equal to $\frac{V^2}{2g}$
- Since laminar flow occurs during the seepage and velocity is very small during laminar flow. Hence in seepage analysis, velocity head may be neglected.

(ii) Pressure head:

- It is equal to $\frac{P}{\gamma_w}$
- If a piezometer or an openstand pipe is inserted at a point of flow. water would stand at a particular height inside the piezometer.
- The actual height of rise of water column in piezometer, represents the pressure head.

(iii) Datum or elevation head:

- It is represented by 'z'
- The elevation or datum head at a point is the vertical distance of that point measured from an assumed datum, Generally, datum is assumed at tail water level.

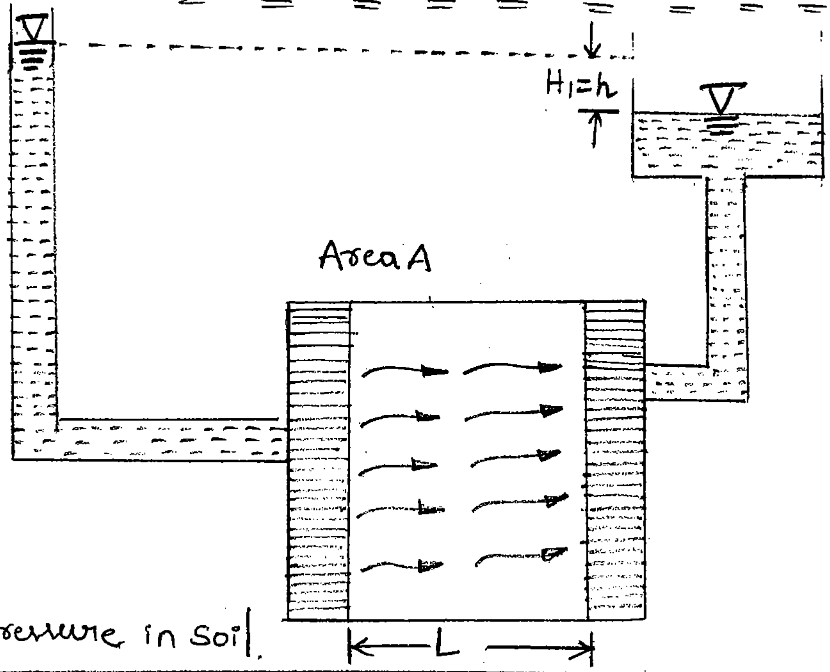
* Total head:

- Total head = velocity head + pressure head + elevation head
- Here, Total head = $0 + \frac{P}{\gamma_w} + z$
- If we insert a stand pipe at a point of flow, the elevation of water level in stand pipe with reference to the datum is equal to total head.
- In seepage analysis, velocity head is neglected. Hence total head and piezometric head both are equal.

Head Loss.

- The difference in total head between two point in a soil through which flow is occuring is represented by the head loss during the flow between these points.
- If flow is occuring through the soil sample, the total head is assumed to reduce during flow.

* Seepage Pressure and its Effect on Effective Stress.



Seepage pressure in soil.

- Seepage pressure:

When water flows through the saturated soil mass, it exerts the pressure over the solids by the virtue of viscous friction and is referred as seepage pressure.

Hence, seepage pressure is the pressure exerted by the water over the soil solids in the mass through which it percolates.

- If 'h' is the hydraulic (i.e Head loss) under which flow is taking place, then seepage is given by

$$P_s = h\gamma_w$$

Also
$$P_s = \frac{h}{z} \cdot z\gamma_w$$

$$P_s = iz\gamma_w$$

If A be the area cross-section, then seepage force is given by

$$P_s = \text{Seepage pressure } (P_s) \times A.$$

$$= iz\gamma_w \times A$$

$$= i(z \times A)\gamma_w$$

$$= iV\gamma_w$$

Seepage per unit volume is known as " Specific seepage force " which is given by

$$P_{ss} = \frac{P_s}{V} = \frac{i.V.\gamma_w}{V} = i\gamma_w.$$

Note:

Seepage pressure always acts in the direction of flow Hence vertical pressure (effective stress) at any given section in flow condition, may either increase or decrease.

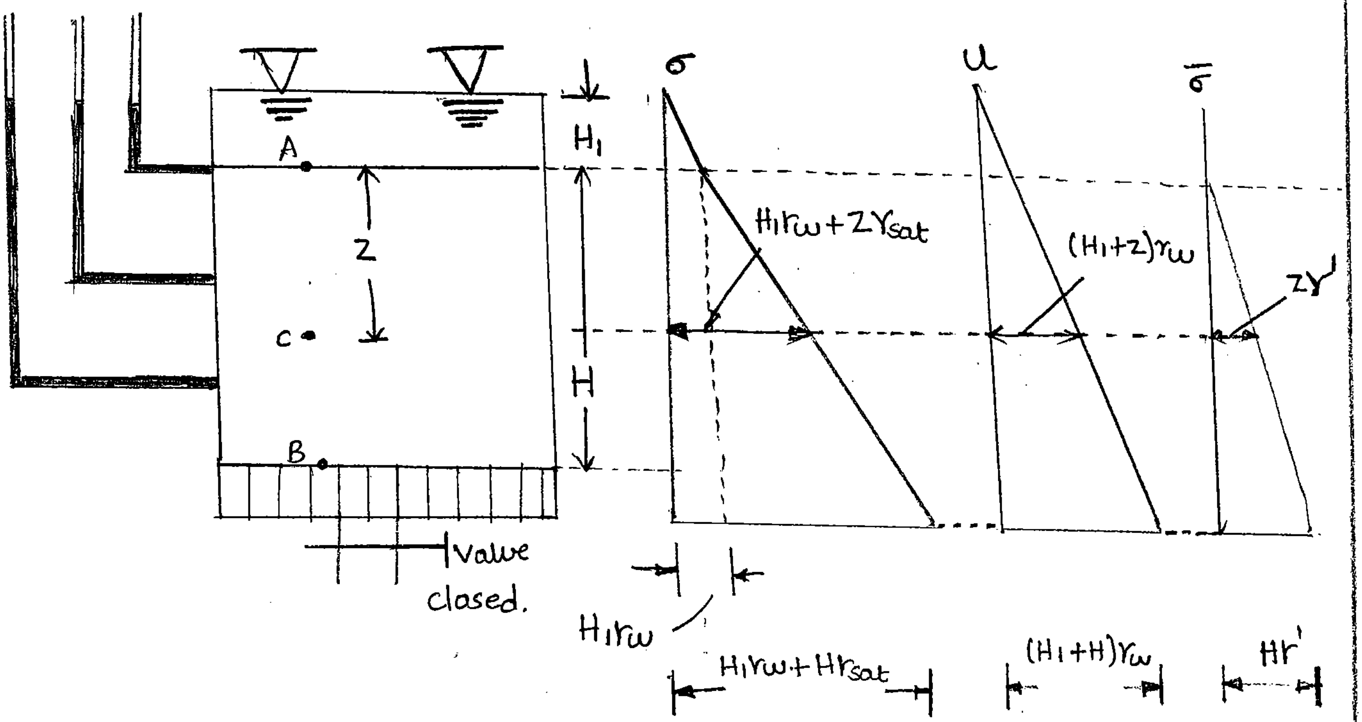
$$\therefore \quad \bar{\sigma}' = \bar{\sigma} \pm P_s$$

Where $\bar{\sigma}$ = effective stress under no flow condition

P_s = Seepage pressure under flow condition.

* No flow condition:

- The figure shows a tank filled with submerged soil. Since the valve at the base is closed, no seepage will occur.



- Pressure head, datum head and total head are tabulated below:

Point	A	B	C
Pressure head	H	H_1+H	H_1+z
Datum head	H	0	$H-z$
Total head	H_1+H	H_1+H	H_1+H.

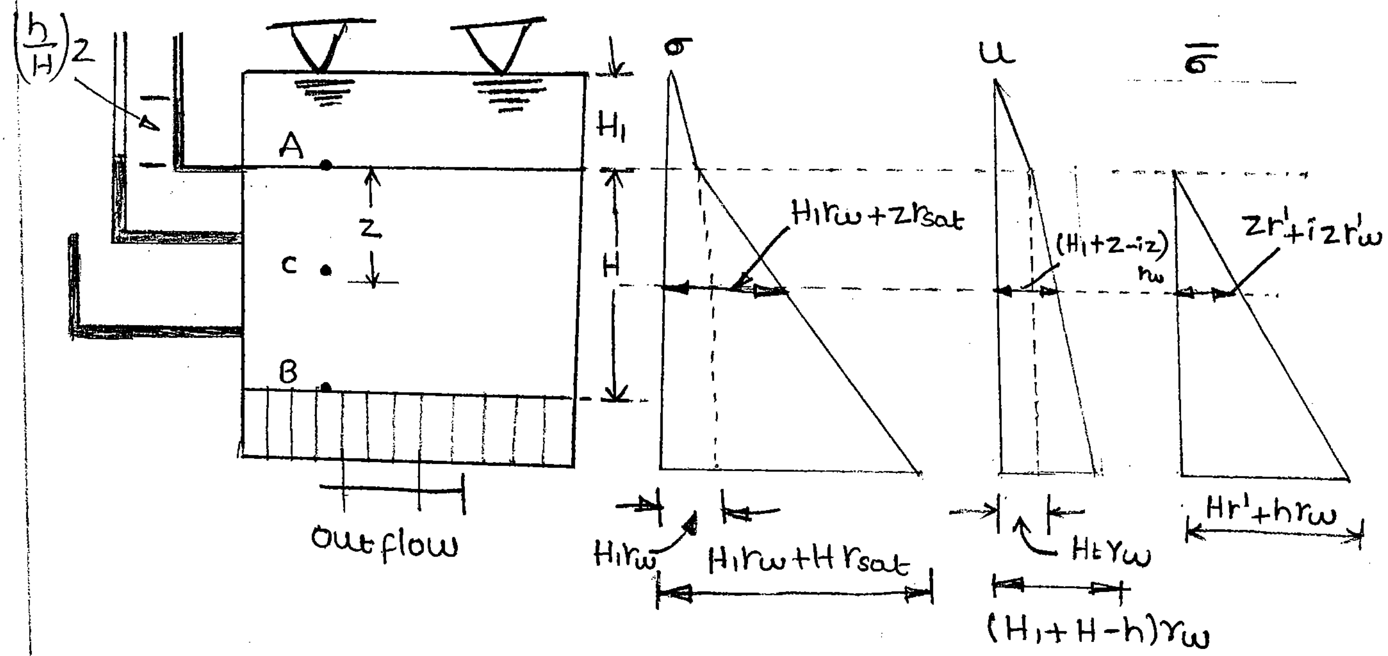
- Total stress, pore water pressure and effective stress and tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$
Pore water pressure (u)	$\gamma_w H_1$	$\gamma_w (H_1 + H)$	$\gamma_w (H_1 + z)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$\gamma_{sat} H + \gamma_w H = \gamma' H$	$\gamma_{sat} z + \gamma_w z = \gamma' z$

The variations of σ, u and $\bar{\sigma}$ are shown in figure.

* Downward Flow

- Figure shows a tank filled with submerged soil, since the valve at the base in open and downward seepage is allowed. The downward seepage increases in effective stress.



Variation of σ, u and $\bar{\sigma}$ with depth downward flow

- Pressure head, datum head and total head are tabulated below,

Point	A	B	C
Pressure Head	H_1	H_1+H-h	H_1+z-iz
Datum head	H	0	$H-z$
Total head	H_1+H	H_1+H-h	H_1+z-iz

Where h = hydraulic head (head loss) under which flow takes place from A to B.; Hydraulic gradient $i = \frac{h}{H}$.

- The total vertical stress at any point in soil mass is due submerged weight of soil mass and seepage pressure depending upon the direction of flow.

- Total stress, pore water pressure and effective stress are tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$	$\gamma_{sat} Z + \gamma_w H_1$
Pore water pressure (u)	$\gamma_w H_1$	$\gamma_w (H_1 + H - h)$	$\gamma_w (H_1 + Z - iZ)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$= (\gamma_{sat} - \gamma_w) H + \gamma_w h$ $= \gamma' H + \gamma_w h$	$= (\gamma_{sat} - \gamma_w) Z + \gamma_w iZ$ $= \gamma' Z + \gamma_w iZ$

The variations of σ, u and $\bar{\sigma}$ are shown in figure above.

* Upward Flow:

- Figure shows, valve at the bottom of tank is open and upward seepage is allowed.

Point	A	B	C
Pressure head	H_1	$H_1 + H + h$	$H_1 + Z + iZ$
Datum head	H	0	$H - Z$
Total Head	$H_1 + H$	$H_1 + H + h$	$H_1 + Z + iZ$

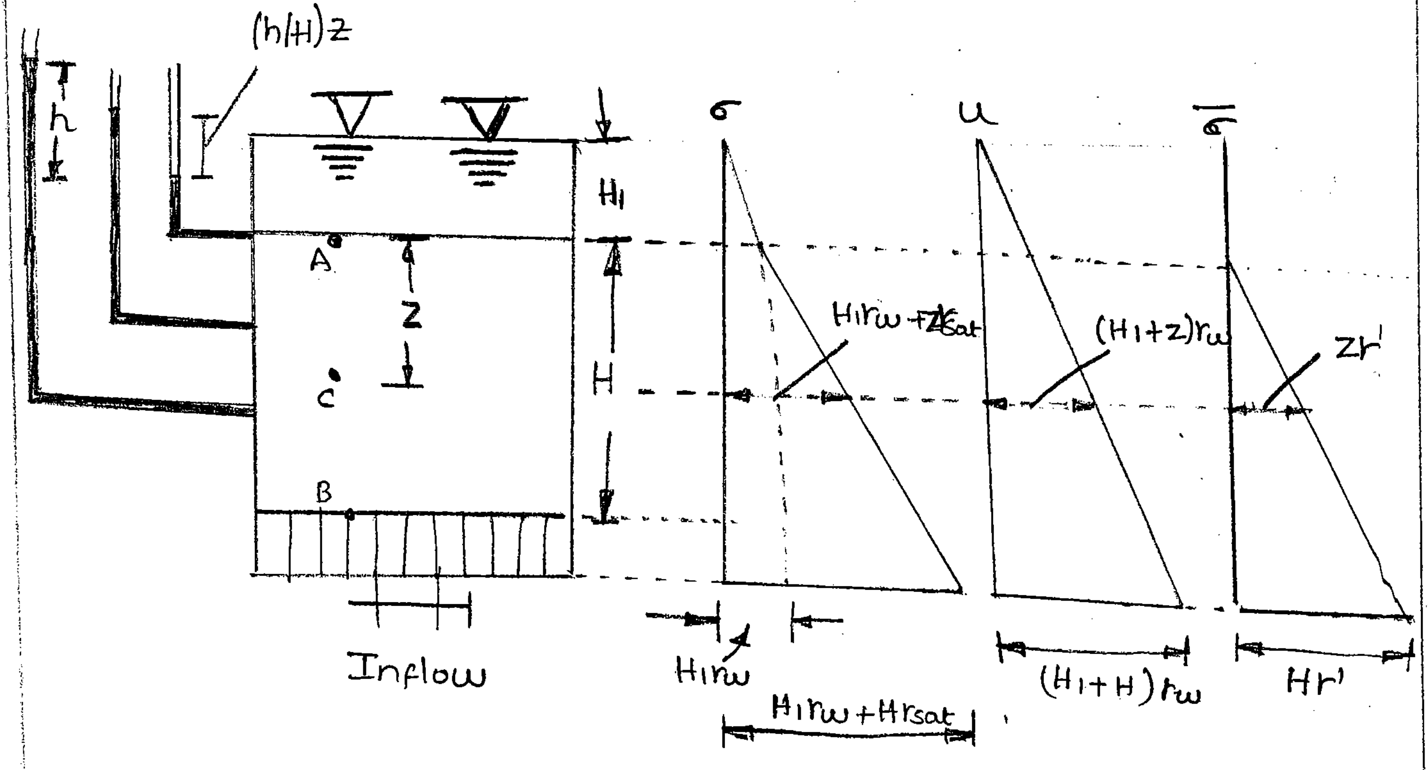
Where h = hydraulic head (headloss) under which flow takes place from A to B

Hydraulic gradient $i = \frac{h}{H}$.

Total stress, Pore water pressure and effective stresses are tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$	$\gamma_{sat} z + \gamma_w H_1$
Pore water pressure (u)	$\gamma_w H_1$	$\gamma_w(H_1+H+h)$	$\gamma_w(H_1+z+iz)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$(\gamma_{sat}-\gamma_w)H + \gamma_w h$ $= \gamma' H + \gamma_w h$ $= \gamma' H - iz\gamma_w$	$(\gamma_{sat}-\gamma_w)H - \gamma_w h$ $= \gamma' H - \gamma_w h$ $= \gamma' H - iz\gamma_w$

The variations of σ, u, and $\bar{\sigma}$ are shown in figure



Variation of σ, u and $\bar{\sigma}$ with depth for upward flow.

Quick Sand Condition

For upward flow condition, effective stress at any point within soilmass is given by

$$\bar{\sigma} = \bar{\sigma} - P_s$$
$$= \gamma' z - P_s$$

It is clear from above equation, upward Seepage pressure decreases effective stresses in the soilmass,

- If the seepage pressure is suchthat it equals the submerged weight of the soilmass, then effective stresses at that location reduced to zero. Under such condition, cohesionless soilmass looses all shear stregth. Now soilmass has a tendency to move also with the flowing water in the upward direction.
- This process in which soil particles are lifted over the soilmass is called quick sand condition. It is also known as "boiling of sand" as the surface of sand looks it is boiling.
- At quick sand condition, net effectivestress is reduce to zero i.e.

$$\bar{\sigma} = 0$$

$$\gamma' z = P_s = 0$$ [Where P_s = Seepage pressure]

$$\gamma' z - i z \gamma_w = 0$$

$$i = \frac{\gamma'}{\gamma_w} = i_{cr}$$

- The hydraulic gradient under which quick sand condition occurs is termed as critical hydraulic gradient. If Void ratio and specific gravity of soil is known, then i_{cr} may be given as

$$i_{cr} = \frac{\gamma'}{\gamma_w} = \frac{\frac{(G-1)\gamma_w}{1+e}}{\gamma_w}$$

$$i_{cr} = \frac{G-1}{1+e}$$

- For fine sand and silts for which specific gravity $G = 2.65$ and void ratio $e \simeq 0.65$

$$i_{cr} = \frac{2.65-1}{1+0.65} \simeq 1$$

- At quick sand condition, cohesionless particles of fine sand may start flowing with the water which may result in piping failure below the hydraulic structure.
- In order to prevent quick sand or piping failure, the hydraulic gradient should be less then critical hydraulic gradient. Hence factor of saftey against quick sand failure or piping failure is

$$F.O.S = \frac{i_c}{i} \qquad \text{Where } i = \frac{h_L}{L}$$

Note:

- Quick sand is not a type of sand. It is a flow condition which exists in cohesionless soil mass where effective stresses are reduced to zero in upward flow conditions

- Quick sand conditions is found only in fine sand and course silts and it is not being observed in the case of gravels, course sands and clays.
- In cohesive soils which posses inherent cohesion, shear-strength is not reduced to zero even when effective stresses are reduced to zero.

For cohesive soils.

$$\tau_1 = c' + \bar{\sigma}\tan\phi'$$

If $P_s = \gamma' z$

then, $\bar{\sigma} = \gamma' z - P_s = 0$

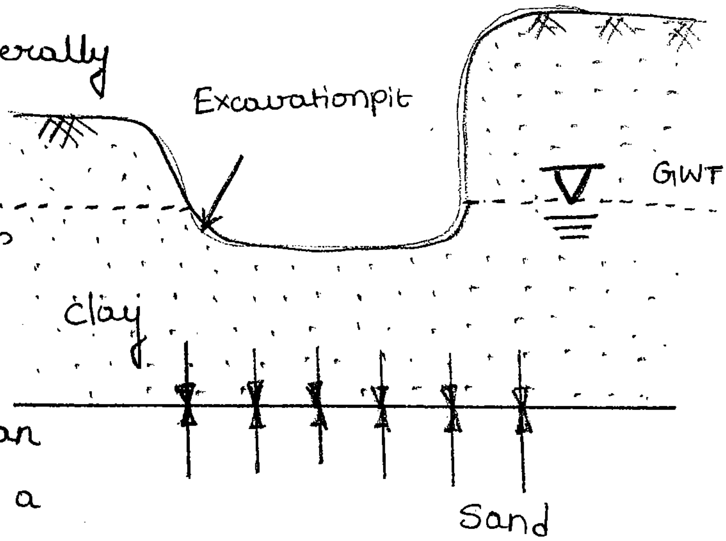
or $\tau_f = c' + 0$

- Quick sand condition is not observed in gravels and coarse sands which are highly permeable soils. As per Darcy's law a large discharge is required to generate critical hydraulic gradient

$$Q = K.i.A \; ; \; i = \frac{Q}{KA}.$$

Remember

Quick sand condition is generally observed when excavation is done below the GWT and water is pumped out to keep the excavated area free from it or it is observed when sand is under artesian pressure and overlain by a clay layer.



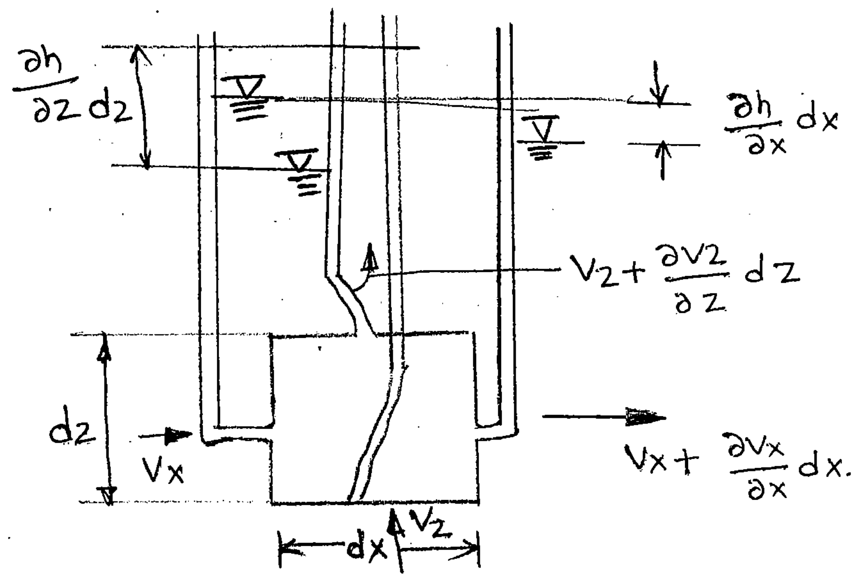
Two - Dimensional Flow - Laplace's Equation

- When flow takes place in 2-D then Darcy's equation cannot be used. In such case of seepage, Laplace equations are used which represents the loss of energy head in any resistive medium like soil.

In the derivation of the Laplace's equation which governs the flow of a fluid through a porous material, assumptions will be used at various stages.

Consider the flow of water into an element in a saturated soil, with dimensions dx and dz in the horizontal and vertical directions in the plane of the paper. The length perpendicular to this plane is dy. Let V be the component of discharge velocity in the horizontal direction, i_x the hydraulic gradient in the horizontal direction $\left(= \frac{\partial h}{\partial x}\right)$, V_z and i_z $\left(= \frac{\partial h}{\partial z}\right)$ the corresponding values in the vertical direction and h the hydraulic head at the element. The velocity in the Y - direction, perpendicular to the plane of the paper, is zero for the two-dimensional flow.

Flow of water into and out of element



The amount of water entering the element in unit time is thus $(V_x dz\, dy + V_z dx\, dy)$ and the amount of water that leaves the element in unit time is

$$\left(V_x dz\, dy + \frac{\partial V_x}{\partial x} dx\, dz\, dy + V_z dx\, dy + \frac{\partial V_z}{\partial z} dz\, dx\, dy\right)$$

In the steady state flow that is being considered, there can be no change in any condition with time. Hence, there can be neither storage nor depletion with time in the amount of water contained in the pores of the soil element. Further with the assumption that the fluid is incompressible and that the volume of voids remains constant during flow, the amount of water entering the element must be equal to the amount of water passing out of the element.

Hence,

$$\left(V_x\, dz\, dy + \frac{\partial v_x}{\partial x} dx\, dz\, dy + V_z\, dx\, dy + \frac{\partial v_z}{\partial z} dz\, dx\, dy\right) -$$

$$(V_x\, dz\, dy + V_z\, dx\, dy) = 0.$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} = 0 \quad (\text{since } dx\,dy\,dz \neq 0).$$

The above equation represents the continuity condition for two dimensional flow (in the x-z plane) under a steady state flow condition.

Assuming the validity of Darcy's law

$$V_x = K_x i_x = K_x \frac{\partial h}{\partial x} \text{ and } V_z = K_z i_z = K_z \frac{\partial h}{\partial z}$$

Again assuming that the soil is homogenous, thereby meaning that K_x and K_z are unique values and are not functions of x and z respectively

$$\frac{\partial V_x}{\partial x} = K_x \frac{\partial^2 h}{\partial x^2} \text{ and } \frac{\partial V_z}{\partial z} = K_z \frac{\partial^2 h}{\partial z^2}.$$

If soil is isotropy with respect to permeability is assumed $K_x = K_z = K$. thus reduces to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0.$$

- The solution of two Laplace equations for the potential and flow functions takes the form of two families of orthogonal curves.

- One set of curve (ψ-lines) represent the trajectories of Seepage and are termed as flowlines. The space between two adjacent flow lines are known as flow channel.

- The another set of curve (ϕ-lines) represents lines of equal head and are termed equipotential lines. The head loss caused by water crossing two adjacent equipotential lines is known as potential drop.

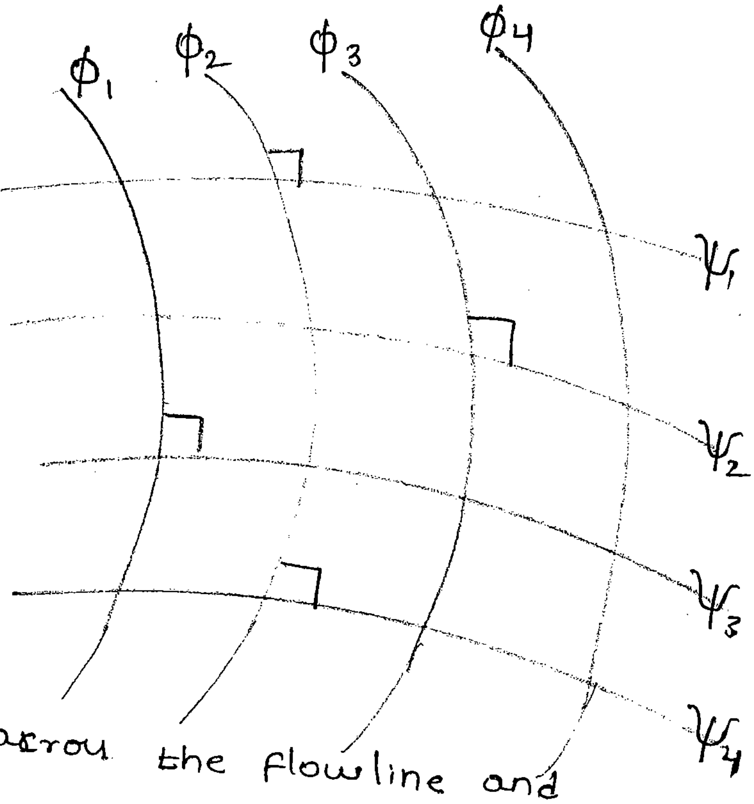
Assumptions made in Laplace Equations:

- Soil is homogenous and fully saturated.
- Flow is Laminar and Darcy's Law is valid
- Pore water and soil solids are assumed to be incompressible.

* Flow Nets:

The entire pattern of flow lines and equipotential lines is referred as a flow net. It is the solution of Laplace's equation for relevant boundary conditions. Thus, a flow net is a graphical representation of the head and direction of seepage at every point.

Properties Of Flow nets



- Flow lines (ψ-lines) and equipotential (ϕ -lines) meet each other orthogonally
- Flow lines and equipotential lines are smooth continous curves, being either elliptical or parabolic in shape.
- There can be no flow across the flow line and velocity of flow is always perpendicular to equipotential-lines.
- Area bounded between two adjacent flow lines is called flow channel or flow path and quantity of water flowing through each channel is same.

- Area bounded between two adjacent equipotential lines and adjacent flow lines is referred to a flow field.

- For isotropic medium, flow field is approximately square which maybe either be linear or curvilinear.

- For non-isotropic medium, flow field is rectangular which may beither be linear or curvilinear.

- Loss of head between two equipotential lines i.e equipotential drop remain constant for all the equipotential lines.

- Flow net remain unchanged if boundary conditions are not altered i.e flow net remain same for the same set of boundary conditions.

Application of FlowNets

Flow net can be used for the determination of Seepage, seepage pressure, hydrostatic pressure and exit gradient.

1. Determination of Seepage

Let Δq be the discharge through each flow channel under the total head of H.

a) For Isotropic medium :

- For isotropic medium, flow fields are square in shape. Let 'bxb' be the size of flow field

$$\therefore \Delta q = K.i.A = K.\frac{\Delta h}{l}(b \times 1)$$

But for isotropic medium $l = b$

$$\therefore \Delta q = K.\Delta h. \quad \text{—— (i)}$$

If 'H' be the total head loss and Nd be the number of potential drops.

then, $\Delta h = \frac{H}{N_d}$

Therefore equation (i) becomes

$$\Delta q = K.\frac{H}{N_d}$$

Let N_f be the total number of flow channels.

The total discharge through the complete flow net per unit length is given as

$$q = \Delta q \times N_f = K.\frac{H}{N_d}.N_f$$

$$= K.H.\frac{N_f}{N_d}.$$

- The ratio $\frac{N_f}{N_d}$ is independent of K and H and is characteristic of the flow net. This is called shape factor of the flow net.

(b) For Non-Isotropic Medium:

- In this case the equation of continuity assumes the form

$$K_x \frac{\partial^2 H}{\partial x^2} + K_y \frac{\partial^2 H}{\partial y^2} = 0.$$

- Solution of above equation cannot be given as it is not in Laplacian form, So we have to transform it by substituting a transformed coordinate x_T for x such that

$$x_T = x\sqrt{\frac{K_y}{K_x}}$$

(or) $$x = x_T \sqrt{\frac{K_x}{K_y}}$$

$$\therefore K_x \frac{\partial^2 H}{\partial^2 x_T \left(\sqrt{\frac{K_x}{K_y}}\right)} + K_y \frac{\partial^2 H}{\partial y^2} = 0$$

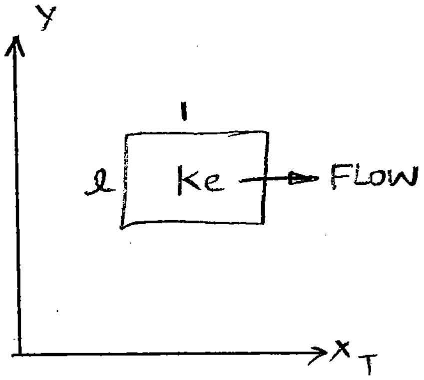
$$K_x \frac{\partial^2 H}{\partial x_T^2} \cdot \frac{K_y}{K_x} + K_y \frac{\partial^2 H}{\partial y^2} = 0.$$

$$\frac{\partial^2 H}{\partial x_T^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

Equation (ii) is in standard laplacian form, flow net for which can be drawn for transformed section. On this transformed scale, flow field will be a square.

Let, equivalent permeability on transformed scale be Ke. Also let Δq_T and Δq_A be the quantity of flow through transformed and actual section respectively.

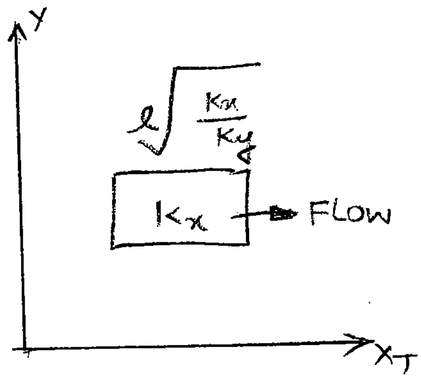
For transformed section



$$\Delta q_T = K_e \frac{\Delta h}{l} (l \times 1)$$

For actual section

$$\Delta q_A = K_x \cdot \frac{\Delta h}{l\sqrt{\frac{K_x}{K_y}}} (l \times 1)$$



But $\Delta q_T = \Delta q_A$

$$\therefore \; K_e \frac{\Delta h}{l} \times 1 = K_x \frac{\Delta h}{l\sqrt{\frac{K_x}{K_y}}} \cdot l$$

$$K_e = \sqrt{K_x K_y}$$

Thus seepage quantity $\Delta q = K_e H \frac{N_f}{N_d}$.

Note: In flow is in 3-D $\quad K_e = \sqrt[3]{K_x \cdot K_y \cdot K_z}$

2. Determination of Seepage Pressure

- Let n_d be the number of potential drop (each of value Δh) by a water particle before reaching a point where seepage pressure is required. Let H be total head governing flow, hence net available head at point under consideration will be.

$$h_1 = H - n_d \Delta h$$

Hence seepage pressure,

$$P_S = h_1 \gamma_w$$

$$P_S = (H - n_d \Delta H)\gamma_w$$

- This pressure acts in the direction of flow.

3. Determination of Uplift Pressure.

- Hydrostatic pressure at any point after 'n_d' equipotential drop is given by

$$u = h_w \gamma_w$$

Where h_w is piezometric head.

$$h_w = h_1 \pm z \text{ or } (H - n_d \Delta H) \pm z$$

Where z is elevation head.

- Generally the downstream water level is usually considered as the datum and all points above the datum are considered as positive.

4. Determination of Exit gradient:

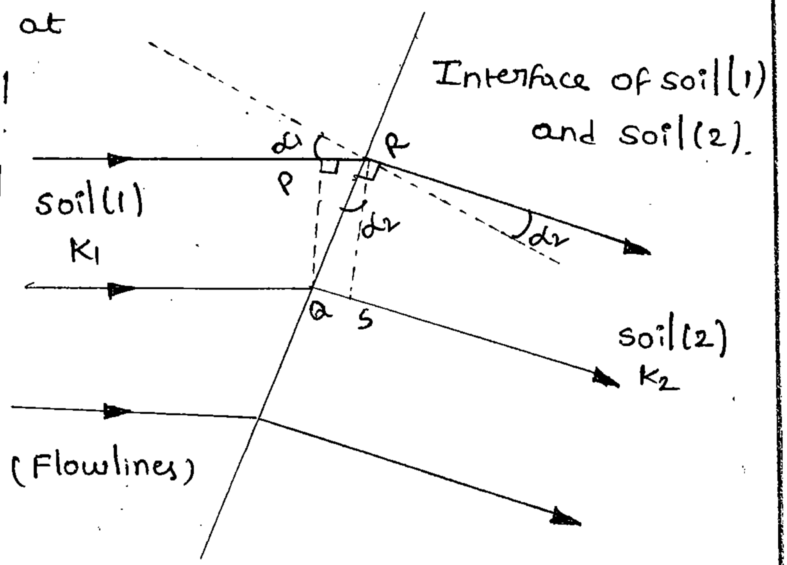
Exit gradient is the hydraulic gradient at the D/s end of the flow line where the percolating water leaves the soil mass and emerges out as free water

$$i_e = \frac{\Delta h}{l}.$$

Where l is Length of the smallest square in the last flow field and Δh is potential drop.

* Flow Through Non-Homogenous section.

- If there is a change in soil conditions, the flow lines are deflected at the interface of the soil with varying permeability k_1 and k_2



Flow through non-homogenous Section

- Let the potential drop from P to Q and from R to S be ΔH

then,

$$\Delta q = k_1\left(\frac{\Delta H}{PR}\right)PQ$$

$$= k_2\left(\frac{\Delta H}{QS}\right)RS.$$

But $\tan\alpha_1 = \dfrac{PR}{PQ}$

$$\alpha_2 = \frac{QS}{RS}$$

$$\therefore \frac{k_1}{\tan\alpha_1} = \frac{k_2}{\tan\alpha_2}$$

Or $\dfrac{k_1}{k_2} = \dfrac{\tan\alpha_1}{\tan\alpha_2}$

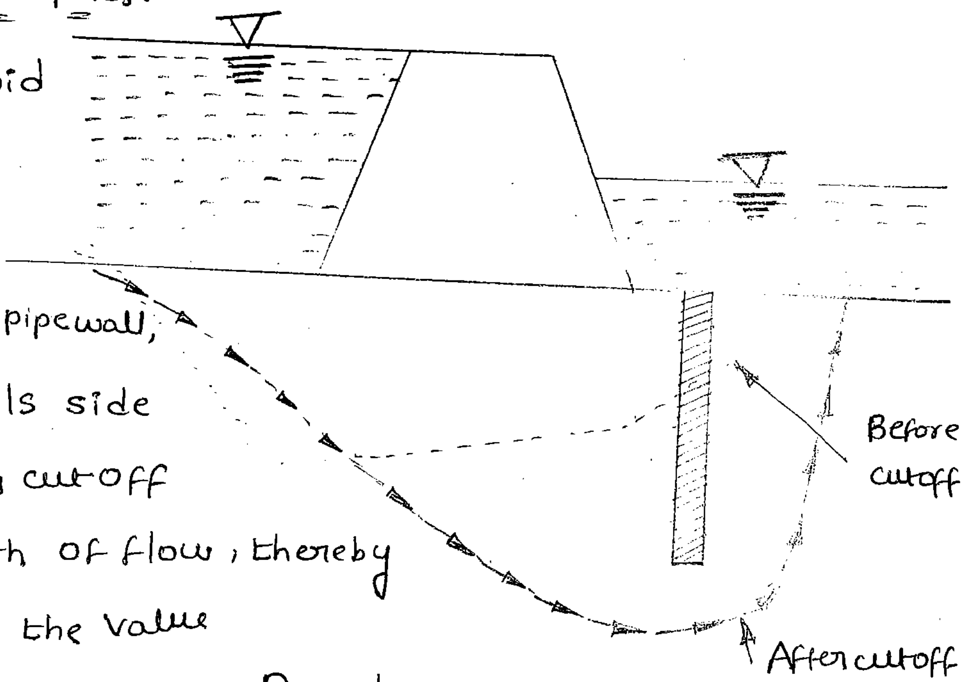
- If $k_1 > k_2$ than $\alpha_1 > \alpha_2$. flow get deflected towards normal.
- If $k_1 < k_2$ than $\alpha_1 < \alpha_2$, flow get deflected away from normal.

* Piping Failure and its protection

- Exit gradient is the governing criteria for the piping failure as it assumes the maximum value amongst all the hydraulic gradient available in the flow net, because adjacent to toe, squares are smallest in size.
- When upward exit gradient approaches critical hydraulic gradient, boiling condition occur, leading to erosion of soil. Such erosion gradually cause a pipe shaped discharge channel. The width of the channel and hydraulic gradient will increase with time and lead to a failure of the structure constructed on or with the soil. Such a mode of failure is called piping.
- For no piping, exit gradient in fine sand is maintained in the range of $\frac{1}{5}$ to $\frac{1}{6}$ and for coarse sand is maintained in the range of $\frac{1}{5}$ to $\frac{1}{4}$.

Methods to prevent Piping Failure

1. Provision of sheet piles:



- In order to avoid piping, sufficient length of vertical cut off i.e sheet pipewall, is provided at D/s side of the dam. This cutoff increases the length of flow, thereby helps in reducing the value of exit gradient.

Providing sheet piles.

2. Provision of Inverted filter:

- Piping can also be avoided by providing protective filters, inverted filters or graded filters. which consist of two or more layers of coarse grained soil applied over less pervious medium.
- These filters help in both avoiding the erosion of the particles of the protective materials and helps in reducing the sufficent head in flow through these filters without the development of excess seepage pressure.

- Specification of filter design as follows

$$\frac{D_{15}\,(\text{filter})}{D_{85}\,(\text{protective material})} < 5$$

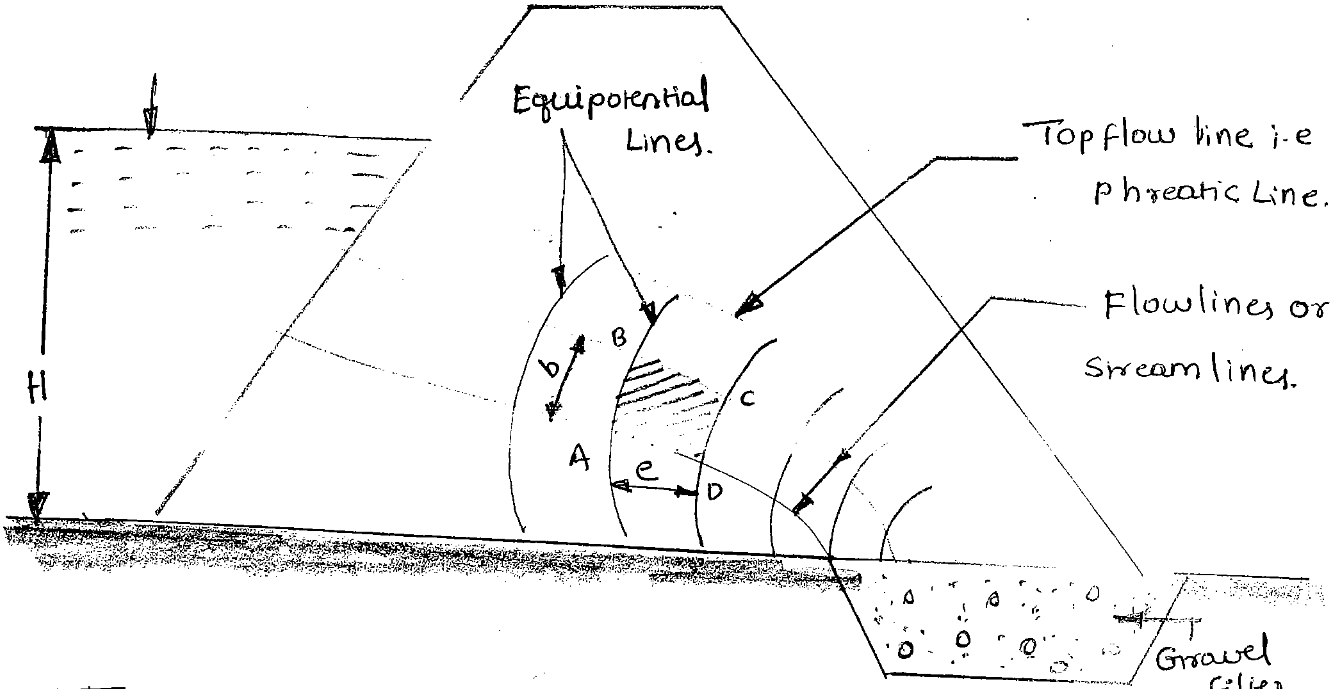
$$4 < \frac{D_{15}\,(\text{filter})}{D_{15}\,(\text{protective material})} < 20$$

$$\frac{D_{50}\,(\text{filter})}{D_{50}\,(\text{protective material})} < 25.$$

- The protective filter permits movement of water but doesnot permit the movement of soil particles. The pore of filter is designed as per condition (i) such that water will enter but soil particle will not enter.

Seepage Through Earthen Dams

- Figure shows a typical section of an earthendam. A typical flow net of seepage through its body is also shown in figure.



- The top most flow Line is called the Line of Seepage or phreatic line. soil mass below the phreatic line is completely saturated whereas soil above is either capillary saturated or partially saturated.
- Pressure above and on the phreatic line is atmospheric and pressure below the phreatic line is hydrostatic
- phreatic line follows the path of base parabola.